Battery Sizing for Grid Connected PV Systems with Fixed Minimum Charging/Discharging Time

Yu Ru, Jan Kleissl, and Sonia Martinez

***Abstract*— In this paper, we study a battery sizing problem for grid-connected photovoltaic (PV) systems assuming that the battery charging/discharging limit scales linearly with its capacity. The objective is to seek a value of the battery size such that the electricity purchase cost from the grid is minimized while satisfying the loads. We propose an upper bound on the storage size, and an algorithm to calculate the exact storage size for the case with ideal PV generation and constant loads; we verify that these are consistent with the results obtained via simulations.**

I. Introduction

Distributed renewable energy systems have seen significant deployment on electric distribution systems to reduce capacity needs and fossil fuel emissions. Among the available renewable energy technologies such as hydroelectric, photovoltaic (PV), wind, geothermal, biomass, and tidal systems, grid-connected solar PV has continued to be the fastest growing distributed power generation technology [1]. Meanwhile, solar energy generation tends to be variable due to the diurnal cycle of the solar geometry and clouds. To complement the PV output during times of peak energy usage and store surplus PV energy for nighttime use, storage devices (such as batteries, ultracapacitors, and pumped hydro storage) can be applied.

In this paper, we study the problem of determining the battery size for grid-connected PV systems. Our setting is shown in Fig. 1. Electrical energy is generated from solar panels, and is used to supply loads. If the PV generation is larger than the loads, extra electrical energy can be stored in batteries to reduce energy imports later on; if the PV generation is smaller than loads, electricity has to be either discharged from the battery or purchased from the grid to meet the loads. Given the high cost of battery storage systems, the size of the battery storage should be small yet minimize electricity purchases from the grid. We thus formulate a battery sizing problem in which the maximum battery charging/discharging rate scales linearly with the battery capacity, and show that there is a unique critical value (denoted as $E^{\text{c}}_{\max}$, refer to Problem 1) such that the cost of electricity purchase remains the same if the net usable battery capacity is larger than or equal to $2E^{\text{c}}_{\max}$ and the cost is strictly larger otherwise. We propose an upper bound on $E^{\text{c}}_{\max}$, and an algorithm to calculate the exact value for the case of typical maximum PV generation on clear days and

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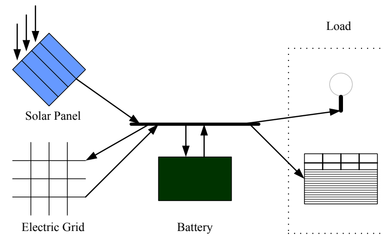


Fig. 1. Grid-connected PV system with battery storage (we assume that the conversion efficiency of DC-to-AC or AC-to-DC converters is 1; therefore, converters are not drawn here).

constant loads; the results are very consistent with the critical value obtained via simulations.

Storage sizing problems for grid-connected systems have been studied extensively, e.g., [2]–[6]. In these studies, batteries are used to reduce the fluctuation of PV output [2], maximize a defined service lifetime/unit cost index of the energy storage system [4], or minimize the cost of the energy storage system [3]. Almost all of these previous studies on storage sizing are based on trial and error approaches except [5], [6]. In [5], the goal is to minimize the long-term average electricity costs in the presence of dynamic pricing as well as investment in storage in a stochastic setting. In [6], we study a battery sizing problem in a setting similar to this work. The key difference is that the maximum charging/discharging rate is fixed in [6], which is relaxed in this work to allow the maximum rate to scale linearly with the battery capacity; this is more realistic because battery cells can be connected in parallel to implement the critical battery capacity.

II. Problem Formulation

A. Photovoltaic Generation

To calculate the electrical power generated from solar panels, we use $P_{\text{pv}}(t) = \text{GHI}(t) \times S \times \eta$, where GHI ($W/m^2$) is the global horizontal irradiation at the location of solar panels, S (m^2) is the total area of solar panels, and η is the conversion efficiency of the solar cells. The PV generation model is a simplified version of the one used in [7].

B. Electric Grid

Electricity can be drawn from (or dumped to) the grid. We associate costs only with the electricity purchase from the grid, and assume that there is no benefit by dumping electricity into the grid. The motivation is that, from a grid operator standpoint, it would be most desirable if PV system served only the local load and did not export to the grid. We use $C_{\text{gp}}(t)$ (¢/kWh) to denote the electricity purchase rate, $P_{\text{gp}}(t)$ (W) to denote the electrical power purchased from the grid at time t, and $P_{\text{gd}}(t)$ (W) to denote the excess electrical power dumped to the grid or curtailed at time t. For simplicity, we assume that $C_{\text{gp}}(t)$ is *time independent* and has the value C_{gp}.

C. Battery

A battery has the dynamic $\frac{dE_B(t)}{dt} = P_B(t)$, where $E_B(t)$ (Wh) is the amount of electrical energy stored in the battery at time t, and $P_B(t)$ (W) is the charging/discharging rate satisfying $P_B(t) > 0$ when charging and $P_B(t) < 0$ when discharging. We impose the following constraints on the battery:

i) $E_{\text{Bmin}} \le E_B(t) \le E_{\text{Bmax}}$, where E_{Bmin} (or E_{Bmax}) is the minimum (or maximum) battery electrical energy (usually E_{Bmin} is chosen to be larger than 0 to prevent fast battery aging), and $0 < E_{\text{Bmin}} \le E_{\text{Bmax}}$, and
ii) $P_{\text{Bmin}} \le P_B(t) \le P_{\text{Bmax}}$, where $-P_{\text{Bmin}} > 0$ (or $P_{\text{Bmax}} > 0$) is the maximum battery discharging (or charging) rate.

We assume that $P_{\text{Bmax}} = -P_{\text{Bmin}} = \frac{E_{\text{Bmax}} - E_{\text{Bmin}}}{T_c}$, where T_c is the minimum time necessary to charge (or discharge) the battery from E_{Bmin} (or E_{Bmax}) to E_{Bmax} (or E_{Bmin}). In other words, the maximum charging/discharging rate scales linearly with the usable capacity of the battery. This assumption is reasonable for scenarios in which the battery is composed of multiple battery cells connected in parallel.

D. Minimization of Electricity Purchase Cost

With all the components introduced earlier, now we can formulate the following problem:

$$
\begin{aligned}
&\min_{P_B, P_{\text{gp}}, P_{\text{gd}}} \int_{t_0}^{t_0+T} C_{\text{gp}} P_{\text{gp}}(\tau) d\tau \\
&\text{s.t. } P_{\text{pv}}(t) + P_{\text{gp}}(t) = P_{\text{gd}}(t) + P_B(t) + P_{\text{load}}(t) \ , \qquad (1)\\
&\tfrac{dE_B(t)}{dt} = P_B(t), \ E_{\text{Bmin}} \le E_B(t) \le E_{\text{Bmax}}, \ E_B(t_0) = E_{\text{Bmin}} \ , \\
&P_{\text{Bmin}} \le P_B(t) \le P_{\text{Bmax}}, \ P_{\text{Bmax}} = -P_{\text{Bmin}} = \tfrac{E_{\text{Bmax}} - E_{\text{Bmin}}}{T_c} \ , \\
&P_{\text{gp}}(t) \ge 0, P_{\text{gd}}(t) \ge 0 \ ,
\end{aligned}
$$

where t_0 is the initial time, T is the time period considered for the cost minimization, and $P_{\text{load}}(t)$ (W) denotes the load at time t. Note that the objective is to minimize the electricity purchase cost from the electric grid during the time interval $[t_0, t_0 + T]$ while guaranteeing that the electricity supply can meet the electricity demand (i.e., the power balance equation (1)) and battery constraints are not violated.

E. Battery Sizing Problem Formulation

Before we formulate the battery sizing problem, we first simplify the cost minimization problem in Section II-D by eliminating variables $P_{\text{gd}}(t)$ and $P_{\text{gp}}(t)$.

Using Eq. (1), we obtain $P_{\text{gp}}(t) = P_{\text{load}}(t) - P_{\text{pv}}(t) + P_B(t) + P_{\text{gd}}(t)$. If $P_{\text{load}}(t) - P_{\text{pv}}(t) + P_B(t) < 0$, we need to choose $P_{\text{gd}}(t) > 0$ to make $P_{\text{gp}}(t) = 0$ so that the cost is minimized at the current time t; if $P_{\text{load}}(t) - P_{\text{pv}}(t) + P_B(t) > 0$, we need to choose $P_{\text{gd}}(t) = 0$ to minimize the electricity purchase costs, and we have $P_{\text{gp}}(t) = P_{\text{load}}(t) - P_{\text{pv}}(t) + P_B(t)$. Therefore, $P_{\text{gp}}(t)$ can be written as $P_{\text{gp}}(t) = \max(0, P_{\text{load}}(t) - P_{\text{pv}}(t) + P_B(t))$ so that the integrand in the objective function is minimized at the current time t. We can plug in the expression of $P_{\text{gp}}(t)$ in the objective function, and obtain an optimization problem which has only one independent variable $P_B(t)$.

We now change variables to make the state variable and the control variable explicit. Let $x(t) = E_B(t) - \frac{E_{\text{Bmax}} + E_{\text{Bmin}}}{2}$, $u(t) = P_B(t)$, and $E_{\text{max}} = \frac{E_{\text{Bmax}} - E_{\text{Bmin}}}{2}$ (note that the division by 2 is to obtain a simple constraint on $x(t)$: $|x(t)| \le E_{\text{max}}$, and $2E_{\text{max}}$ is the net (usable) battery capacity; thus, we can equivalently use E_{max} as the battery capacity). Then we have

$$
P_{\text{Bmax}} = -P_{\text{Bmin}} = \frac{2E_{\text{max}}}{T_c} \ , \qquad (2)
$$

and can rewrite the optimization problem as

$$
\begin{aligned}
J = &\min_u \int_{t_0}^{t_0+T} C_{\text{gp}} \max(0, P_{\text{load}}(\tau) - P_{\text{pv}}(\tau) + u(\tau)) d\tau \\
&\text{s.t. } \frac{dx(t)}{dt} = u(t), \ |x(t)| \le E_{\text{max}}, \ x(t_0) = -E_{\text{max}} \ , \\
&P_{\text{Bmin}} \le u(t) \le P_{\text{Bmax}}, \ P_{\text{Bmax}} = -P_{\text{Bmin}} = \tfrac{2E_{\text{max}}}{T_c} \ . \qquad (3)
\end{aligned}
$$

We define the set of feasible controls (denoted as U_{feasible}) as controls that satisfy the constraints $-\frac{2E_{\text{max}}}{T_c} \le u(t) \le \frac{2E_{\text{max}}}{T_c}$.

If we *fix* the parameters t_0, T, and T_c, J is a function of E_{max}, which is denoted as $J(E_{\text{max}})$. If $E_{\text{max}} = 0$, then $u(t) = 0$, and J reaches the largest value $J_{\text{max}} = \int_{t_0}^{t_0+T} C_{\text{gp}} \max(0, P_{\text{load}}(\tau) - P_{\text{pv}}(\tau)) d\tau$. If we increase E_{max}, intuitively J will decrease (though may not strictly decrease) because the battery can be utilized to store extra electrical energy generated from PV to be potentially used later on when the load exceeds the PV generation. This is formally justified by the following lemma.

Lemma 1 Given the optimization problem in Eq. (3) with fixed t_0, T, and T_c, if $E^1_{\text{max}} < E^2_{\text{max}}$, then $J(E^1_{\text{max}}) \ge J(E^2_{\text{max}})$.

Proof: Given E^1_{max}, suppose a feasible control $u^1(t)$ achieves the minimum electricity purchase cost $J(E^1_{\text{max}})$, and the corresponding state x is $x^1(t)$. Since $|x^1(t)| \le E^1_{\text{max}} < E^2_{\text{max}}$ and $|u^1(t)| \le \frac{2E^1_{\text{max}}}{T_c} < \frac{2E^2_{\text{max}}}{T_c}$, $u^1(t)$ is also a feasible control for problem (3) with the state constraint E^2_{max}, and results in the cost $J(E^1_{\text{max}})$. Since $J(E^2_{\text{max}})$ is the minimal cost over the set of all feasible controls which include $u^1(t)$, we must have $J(E^1_{\text{max}}) \ge J(E^2_{\text{max}})$. ■

In other words, J is decreasing with respect to the parameter E_{max}, and is lower bounded by 0. We are interested in finding the smallest value of E_{max} (denoted as $E^{\text{c}}_{\text{max}}$) such that J remains the same for any $E_{\text{max}} \ge E^{\text{c}}_{\text{max}}$.

Problem 1 (**Storage Size Determination**) Given the optimization problem in Eq. (3) with fixed t_0, T, and T_c, determine a critical value $E^{\text{c}}_{\text{max}} \ge 0$ such that i) $\forall E_{\text{max}} < E^{\text{c}}_{\text{max}}$, $J(E_{\text{max}}) > J(E^{\text{c}}_{\text{max}})$, and ii) $\forall E_{\text{max}} \ge E^{\text{c}}_{\text{max}}$, $J(E_{\text{max}}) = J(E^{\text{c}}_{\text{max}})$.

Note that the critical value $E^{\text{c}}_{\text{max}}$ is bounded (refer to Proposition 1), and unique as stated below (which can be proved via contradiction based on the definition of $E^{\text{c}}_{\max}$).

Lemma 2 Given the optimization problem in Eq. (3) with fixed t_0, T, and T_c, $E^{\text{c}}_{\max}$ is unique.

One may try to calculate $E^{\text{c}}_{\max}$ by first obtaining an explicit expression for $J(E_{\max})$ and then solve for $E^{\text{c}}_{\max}$. However, the optimization problem in Eq. (3) is difficult to solve due to the state and control constraints. Instead, we first focus on bounding the critical value $E^{\text{c}}_{\max}$ in the next section, and then study the problem for specific scenarios in Section IV.

III. Upper Bound on $E^{\text{c}}_{\max}$

To make use of the battery, we impose the following assumption on Problem 1, which guarantees that the battery can be charged at t_1 and the battery discharging at $t_2 > t_1$ can strictly reduce the cost.

Assumption 1 There exist $t_1, t_2 \in [t_0, t_0 + T]$, such that $t_1 < t_2$, $P_{\text{pv}}(t_1) - P_{\text{load}}(t_1) > 0$ and $P_{\text{pv}}(t_2) - P_{\text{load}}(t_2) < 0$.

Proposition 1 Given the optimization problem in Eq. (3) with fixed t_0, T, and T_c under Assumption 1,

$$0 < E^{\text{c}}_{\max} \leq \max\left(\frac{\min(A,B)}{2}, \frac{\max(C,D)T_c}{2}\right) ,$$

where

$$A = \int_{t_0}^{t_0+T} \max(0, P_{\text{pv}}(t) - P_{\text{load}}(t))dt \ , \tag{4}$$

$$B = \int_{t_0}^{t_0+T} \max(0, P_{\text{load}}(t) - P_{\text{pv}}(t))dt \ . \tag{5}$$

$$C = \max_{t\in[t_0,t_0+T]} (P_{\text{load}}(t) - P_{\text{pv}}(t)) \ , \tag{6}$$

$$D = \max_{t\in[t_0,t_0+T]} (P_{\text{pv}}(t) - P_{\text{load}}(t)) \ . \tag{7}$$

Proof: We first show $E^{\text{c}}_{\max} > 0$ via contradiction. Since $E^{\text{c}}_{\max} \geq 0$, we need exclude the case $E^{\text{c}}_{\max} = 0$. Suppose $E^{\text{c}}_{\max} = 0$. If we choose $E_{\max} > E^{\text{c}}_{\max} = 0$, $J(E_{\max}) < J(E^{\text{c}}_{\max})$ because under Assumption 1 a battery can store the extra PV generated electrical energy first and then use it later on to strictly reduce the cost compared with the case without a battery (i.e., the case with $E_{\max} = 0$). A contradiction to the definition of $E^{\text{c}}_{\max}$.

To show $E^{\text{c}}_{\max} \leq \max(\frac{\min(A,B)}{2}, \frac{\max(C,D)T_c}{2})$, it is sufficient to show that if $E_{\max} \geq \max(\frac{\min(A,B)}{2}, \frac{\max(C,D)T_c}{2})$, then $J(E_{\max}) = J(\max(\frac{\min(A,B)}{2}, \frac{\max(C,D)T_c}{2}))$.

Before we prove the result, we briefly discuss the meanings of A, B, C, D. In Eq. (4), $\max(0, P_{\text{pv}}(t) - P_{\text{load}}(t))$ is the extra PV generated electrical power after supplying the load, and therefore, A is the maximum total amount of electrical energy that can be stored in the battery during the interval $[t_0, t_0 + T]$ without considering the maximum charging rate P_{Bmax}. In Eq. (5), $\max(0, P_{\text{load}}(t) - P_{\text{pv}}(t))$ is the electrical power strictly necessary to supply the load at time t via either battery discharging or grid purchase. Therefore, B is the total amount of electrical energy that needs to be discharged from the battery or purchased from the grid to meet the load. Since the objective is to minimize the electricity purchase from the grid, we would like to fulfill B using the electrical energy discharged from the battery (whenever possible). The actual discharged electrical energy can be smaller than B if the maximum discharging rate $-P_{\text{Bmin}}$ is smaller than $\max(0, P_{\text{load}}(t) - P_{\text{pv}}(t))$ for some t. In Eq. (6), C is the minimum of the battery discharging rate so that battery discharging and PV generation can meet the load. In Eq. (7), D is the minimum of the battery charging rate so that the battery can be charged whenever there is extra PV generated power after supplying the load.

We first consider $\frac{\min(A,B)}{2} \geq \frac{\max(C,D)T_c}{2}$. In this case, the upper bound becomes $\frac{\min(A,B)}{2}$, and $E_{\max} \geq \frac{\min(A,B)}{2} \geq \frac{\max(C,D)T_c}{2}$. Note that $E_{\max} \geq \frac{\max(C,D)T_c}{2}$ implies that $P_{\text{Bmax}} = -P_{\text{Bmin}} = \frac{2E_{\max}}{T_c} \geq \max(C, D)$. Therefore, the maximum battery charging/discharging rate is large enough to ensure that PV generated extra power can be stored (as long as the battery capacity is large enough) and the battery discharging can meet the demand at any time (as long as there is enough electrical energy stored). Thus, in this case, the only factor that matters is the electrical energy stored and the electrical energy that can be discharged. If $E_{\max} \geq \frac{\min(A,B)}{2}$, the battery is large enough to store all extra PV generated electrical energy (when $A \leq B$) or supply the load all the time (when $A > B$). Then it is not possible to lower the cost with an $E_{\max} \geq \frac{\min(A,B)}{2}$, which implies that $J(E_{\max}) = J(\frac{\min(A,B)}{2})$.

We now consider $\frac{\min(A,B)}{2} < \frac{\max(C,D)T_c}{2}$. In this case, the upper bound becomes $\frac{\max(C,D)T_c}{2}$, and $E_{\max} \geq \frac{\max(C,D)T_c}{2} > \frac{\min(A,B)}{2}$, i.e., the battery capacity is large enough to hold all extra PV generated electrical energy (when $A \leq B$) or electrical energy necessary to power the load (when $A > B$). Thus, in this case, the only factor that matters is the maximum charging/discharging rate. If $E_{\max} \geq \frac{\max(C,D)T_c}{2}$, then $P_{\text{Bmax}} = -P_{\text{Bmin}} \geq \max(C, D)$. In other words, the maximum battery charging/discharging rate is large enough to ensure that PV generated extra electrical power can be stored and the battery discharging can meet the demand at any time. Therefore, it is not possible to lower the cost, which implies that $J(E_{\max}) = J(\frac{\max(C,D)T_c}{2})$. This completes the proof. ■

IV. Ideal PV Generation and Constant Load

In this section, we study how to obtain the critical value for the scenario in which the PV generation is ideal and the load is constant as specified below.

Assumption 2 The initial time t_0 is 0000 h LST, $T = k \times 24(h)$ where k is a nonnegative integer, $P_{\text{pv}}(t)$ is periodic on a timescale of 24 hours, and satisfies the following property for $t \in [0, 24(h)]$: there exist three time instants $0 < t_{\text{sunrise}} < t_{\max} < t_{\text{sunset}} < 24(h)$ such that

- $P_{\text{pv}}(t) = 0$ for $t \in [0, t_{\text{sunrise}}] \cup [t_{\text{sunset}}, 24(h)]$;
- $P_{\text{pv}}(t)$ is continuous and strictly increasing (or decreasing) for $t \in [t_{\text{sunrise}}, t_{\max}]$ (or $t \in [t_{\max}, t_{\text{sunset}}]$);
- $P_{\text{pv}}(t)$ achieves its maximum $P^{\max}_{\text{pv}}$ at $t_{\max}$,

and $P_{\text{load}}(t) = P_{\text{load}}$ for $t \in [t_0, t_0 + T]$, where P_{load} is a constant satisfying $0 < P_{\text{load}} < P^{\max}_{\text{pv}}$.

It can be verified that Assumption 2 implies Assumption 1.

A. $E^{\rm c}_{\rm max}$ under Fixed Maximum Charging/Discharging Rate

If $P_{\rm Bmin}$ and $P_{\rm Bmax}$ are fixed, we have the following results regarding $E^{\rm c}_{\rm max}$.

Proposition 2 [6] Given the optimization problem in Eq. (3) with fixed $t_0, T, P_{\rm Bmin}$ and $P_{\rm Bmax}$ under Assumption 2. **I)** If $T = 24(h)$, $E^{\rm c}_{\rm max} = \frac{\min(A_1,B_1)}{2}$, where

$$A_1 = \int_0^{24} \min(P_{\rm Bmax}, \max(0, P_{\rm pv}(t) - P_{\rm load}))dt \ , \quad (8)$$

$$B_1 = \int_{t_{\rm max}}^{24} \min(-P_{\rm Bmin}, \max(0, P_{\rm load} - P_{\rm pv}(t)))dt \ ; \quad (9)$$

II) If $T = k \times 24(h)$ where $k > 1$ is a positive integer, $E^{\rm c}_{\rm max} = \frac{\min(A_2,B_2)}{2}$, where $A_2 = A_1$ and

$$B_2 = \int_{t_{\rm max}}^{t_{\rm max}+24} \min(-P_{\rm Bmin}, \max(0, P_{\rm load} - P_{\rm pv}(t)))dt \ . \quad (10)$$

Remark 1 Note that if $T = 24(h)$, A_1 is the amount of extra PV generated electrical energy that can be stored in a battery, and B_1 is the amount of electrical energy that is necessary to supply the load *and* can be provided by battery discharging. ■

Instead, if we fix the charging time T_c, then $P_{\rm Bmin}$ and $P_{\rm Bmax}$ change as $E_{\rm max}$ changes due to Eq. (2). Based on Proposition 2, one might try to calculate $E^{\rm c}_{\rm max}$ by replacing $P_{\rm Bmax}$ and $P_{\rm Bmin}$ in Proposition 2 with Eq. (2) and then solving $E^{\rm c}_{\rm max} = \frac{\min(A_1,B_1)}{2}$ as an implicit equation of $E^{\rm c}_{\rm max}$. Numerically we can solve the implicit equation; however, the obtained $E^{\rm c}_{\rm max}$ may not be consistent with the value obtained via simulations for certain T_c, which is illustrated in Section V-B. Therefore, new techniques are necessary to handle the scenario with fixed T_c.

B. $E^{\rm c}_{\rm max}$ under Fixed Minimum Charging/Discharging Time

Now we study how to calculate $E^{\rm c}_{\rm max}$ when T_c is fixed. This calculation is based on the analysis of Proposition 2, its application in the current setting, and insights provided by simulations.

We first consider the scenario in which $P_{\rm load} < P^{\rm max}_{\rm pv} - P_{\rm load}$ and $T = 24(h)$. As illustrated in the simulation Section V-B, the value of $E^{\rm c}_{\rm max}$ given by Proposition 2 is correct for certain values of T_c. Therefore, we begin our analysis based on the result in Proposition 2.

For the quantity A_1 defined in Eq. (8), the maximum value of $\max(0, P_{\rm pv}(t) - P_{\rm load})$ is $P^{\rm max}_{\rm pv} - P_{\rm load}$. Therefore, if $P_{\rm Bmax} \geq P^{\rm max}_{\rm pv} - P_{\rm load}$, A_1 becomes

$$A_1 = \int_0^{24} \max(0, P_{\rm pv}(t) - P_{\rm load})dt \ , \quad (11)$$

which does not directly depend on $E_{\rm max}$ (recall $P_{\rm Bmax} = \frac{2E_{\rm max}}{T_c}$). Similarly, If $-P_{\rm Bmin} \geq P_{\rm load}$, B_1 becomes

$$B_1 = \int_{t_{\rm max}}^{24} \max(0, P_{\rm load} - P_{\rm pv}(t))dt \ , \quad (12)$$

which does not directly depend on $E_{\rm max}$ either (recall $-P_{\rm Bmin} = \frac{2E_{\rm max}}{T_c}$).

Now we study different ranges for $P_{\rm Bmax}$ under the constraint $P_{\rm load} < P^{\rm max}_{\rm pv} - P_{\rm load}$. There are three possibilities:

i) $P_{\rm Bmax} < P_{\rm load}$, or equivalently, $E_{\rm max} < E^{\rm L}_{\rm max}$, where $E^{\rm L}_{\rm max} = \frac{P_{\rm load}T_c}{2}$. In this case, it means that $-P_{\rm Bmin} = P_{\rm Bmax}$ is not large enough to supply the demand from the load for $t \in [t_{\rm sunset}, 24]$, which results in grid purchase. By increasing the battery capacity (which increases $P_{\rm Bmax}$), the electricity purchased from the grid will be lowered (thus, also lowering the cost) because more extra PV generation can be stored due to $P_{\rm Bmax} < P_{\rm load} < P^{\rm max}_{\rm pv} - P_{\rm load}$. Therefore, $E^{\rm c}_{\rm max}$ cannot be smaller than $E^{\rm L}_{\rm max}$.

ii) $P_{\rm load} \leq P_{\rm Bmax} < P^{\rm max}_{\rm pv} - P_{\rm load}$, or equivalently, $E^{\rm L}_{\rm max} \leq E_{\rm max} < E^{\rm H}_{\rm max}$, where $E^{\rm H}_{\rm max} = \frac{(P^{\rm max}_{\rm pv} - P_{\rm load})T_c}{2}$. In this case, the quantity B_1 is given in Eq. (12) but the quantity A_1 is still given in Eq. (8). Based on the meanings of A_1 and B_1 in Remark 1, to minimize the grid purchase cost, we need to make sure $A_1 \geq B_1$ and $2E_{\rm max} \geq B_1$; in other words, the supply is at least as large as the demand and the battery can store the needed electrical energy B_1. By increasing $E_{\rm max}$ starting from $E^{\rm L}_{\rm max}$, A_1 will strictly monotonically increase based on Eq. (8), and therefore, $A_1 \geq B_1$ will eventually be satisfied (though $E_{\rm max}$ may increase even beyond $E^{\rm H}_{\rm max}$). Therefore, the smallest value of $E_{\rm max}$ satisfying $E^{\rm L}_{\rm max} \leq E_{\rm max} < E^{\rm H}_{\rm max}$, $A_1 \geq B_1$, and $2E_{\rm max} \geq B_1$, will be the critical value $E^{\rm c}_{\rm max}$, which can be proved via contradiction. If no such value can be found, we need to consider $E_{\rm max} \geq E^{\rm H}_{\rm max}$.

iii) $P^{\rm max}_{\rm pv} - P_{\rm load} \leq P_{\rm Bmax}$, or equivalently, $E_{\rm max} \geq E^{\rm H}_{\rm max}$. In this case, the quantity A_1 is given in Eq. (11) and B_1 is given in Eq. (12). Based on the meanings of A_1, B_1, and the result in Proposition 2, the battery capacity can be chosen to be $\frac{\min(A_1,B_1)}{2}$. However, we also need to guarantee that $E_{\rm max} \geq E^{\rm H}_{\rm max}$. Therefore, $\max(E^{\rm H}_{\rm max}, \frac{\min(A_1,B_1)}{2})$ is the critical value, which can be proved via contradiction.

If $P_{\rm load} \geq P^{\rm max}_{\rm pv} - P_{\rm load}$, we can redefine $\frac{P_{\rm load}T_c}{2}$ as $E^{\rm H}_{\rm max}$, and $\frac{(P^{\rm max}_{\rm pv} - P_{\rm load})T_c}{2}$ as $E^{\rm L}_{\rm max}$. Similar analysis can be applied to this scenario and the difference lies in the case $E^{\rm L}_{\rm max} \leq E_{\rm max} < E^{\rm H}_{\rm max}$. In this case, it might not be possible that $A_1 \geq B_1$ due to larger loads; when $A_1 < B_1$, we require $2E_{\rm max} \geq A_1$, i.e., when the supply cannot meet the demand, the battery should store all extra PV generated electrical energy to lower the grid purchase.

When $T = k \times 24$ for $k > 1$, we can do the same analysis based on the result in part **II** of Proposition 2. Now an algorithm to calculate the value $E^{\rm c}_{\rm max}$ is given in Algorithm 1. The inputs are $t_0, T, T_c, P_{\rm load}, P_{\rm pv}(t)$, and the output is $E^{\rm c}_{\rm max}$. The structure of the algorithm follows the previous discussion: Steps 1-2 set $E^{\rm L}_{\rm max}$ and $E^{\rm H}_{\rm max}$; Steps 3-7 deal with the case in which $E^{\rm L}_{\rm max} \leq E_{\rm max} < E^{\rm H}_{\rm max}$ while Steps 10-12 handle the case in which $E_{\rm max} \geq E^{\rm H}_{\rm max}$. Based on the expression for $E^{\rm c}_{\rm max}$ at Step 11, it can be verified that the upper bound in Proposition 1 also holds in the current

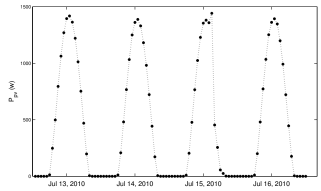


Fig. 2. PV output for July 13 - 16, 2010, at La Jolla, California.

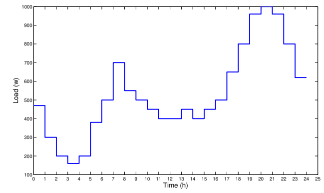


Fig. 3. A typical residential load profile.

setting.

Algorithm 1 Calculation of $E^{\text{c}}_{\text{max}}$

Input: Fixed $t_0, T, T_c, P_{\text{load}}$, and $P_{\text{pv}}(t)$ for $t \in [t_0, t_0 + T]$ satisfying Assumption 2

Output: $E^{\text{c}}_{\text{max}}$

Set $E^{\text{L}}_{\text{max}} = \frac{\min(P_{\text{load}},\ P^{\text{max}}_{\text{pv}} - P_{\text{load}}) \times T_c}{2}$;
Set $E^{\text{H}}_{\text{max}} = \frac{\max(P_{\text{load}},\ P^{\text{max}}_{\text{pv}} - P_{\text{load}}) \times T_c}{2}$;
if $P_{\text{load}} < P^{\text{max}}_{\text{pv}} - P_{\text{load}}$ **then**
  Find the smallest value of E_{max} satisfying $E^{\text{L}}_{\text{max}} \leq E_{\text{max}} < E^{\text{H}}_{\text{max}}$ and:
    i) $A_1 \geq B_1$ and $2E_{\text{max}} \geq B_1$ for $T = 24$;
    ii) $A_2 \geq B_2$ and $2E_{\text{max}} \geq B_2$ for $T = k \times 24$;
else
  Find the smallest value of E_{max} satisfying $E^{\text{L}}_{\text{max}} \leq E_{\text{max}} < E^{\text{H}}_{\text{max}}$ and:
    i) either $A_1 \geq B_1$ and $2E_{\text{max}} \geq B_1$, or $A_1 < B_1$ and $2E_{\text{max}} \geq A_1$ for $T = 24$;
    ii) either $A_2 \geq B_2$ and $2E_{\text{max}} \geq B_2$, or $A_2 < B_2$ and $2E_{\text{max}} \geq A_2$ for $T = k \times 24$;
end if
if a solution is found **then**
  Set $E^{\text{c}}_{\text{max}}$ as the smallest value of E_{max};
else
  i) If $T = 24$, calculate A_1, B_1, and set $E^{\text{c}}_{\text{max}} = \max(E^{\text{H}}_{\text{max}}, \frac{\min(A_1, B_1)}{2})$;
  ii) If $T = k \times 24$, calculate A_2, B_2, and set $E^{\text{c}}_{\text{max}} = \max(E^{\text{H}}_{\text{max}}, \frac{\min(A_2, B_2)}{2})$;
end if
Output $E^{\text{c}}_{\text{max}}$.

V. Simulations

In this section, we verify the results in Sections III and IV via simulations. The parameters used in Section II are chosen based on a typical residential home setting. The GHI data is the measured GHI between July 13 and July 16, 2010 at La Jolla, California. In our simulations, we use $\eta = 0.15$, and $S = 10(m^2)$. Thus $P_{\text{pv}}(t) = 1.5 \times \text{GHI}(t)(W)$. We choose t_0 as 0000 h LST on Jul 13, 2010, and the hourly PV output is given in Fig. 2 for the following four days starting from t_0. The PV generation roughly satisfies Assumption 2. Note that $0 \leq P_{\text{pv}}(t) < 1500(W)$ for $t \in [t_0, t_0 + 96]$.

We set $C_{\text{gp}} = 7.8$¢$/kWh$, which is the semipeak rate for the summer season proposed by SDG&E (San Diego Gas & Electric) [8]. For the battery, we choose $E_{\text{Bmin}} = 0.4 \times E_{\text{Bmax}}$, and then $E_{\text{max}} = 0.3 \times E_{\text{Bmax}}$. T_c is chosen to be between $2(h)$ and $14(h)$ with the step size $1(h)$. We choose P_{load} to be between 0 and $1500(W)$, and the load is used from t_0 to $t_0 + T$ to satisfy Assumption 2. To run simulations, we discretize the battery dynamic equation in Section II-C as $E_B(k+1) = E_B(k) + P_B(k)$ using one hour as the sampling interval.

A. Dynamic Loads

We first examine the upper bound in Proposition 1 using dynamic loads. The load profile for one day is given in Fig. 3, which resembles the residential load profile in Fig. 8(b) in [7]. For multiple day simulations, the same load profile is used for all days. We study how the cost function J changes as E_{max} varying from 0 to $6000(Wh)$ with the step size $10(Wh)$, by solving the optimization problem in Eq. (3) via linear programming using the CPlex sover [9]. If $T = 24(h)$ and $T_c = 2(h)$ (or $T_c = 7(h)$, respectively), the plot of the minimum costs versus E_{max} is given in Fig. 4(a) (or (b), respectively). The plots confirm the result in Lemma 1, and also show the existence of the unique $E^{\text{c}}_{\text{max}}$. The corresponding $E^{\text{c}}_{\text{max}}$ values are shown in the column "Sim" with $T = 24(h)$ in Table I, which can be identified from Fig. 4. The upper bounds in Proposition 1 are shown in the column "UB" with $T = 24(h)$ in Table I. Similarly, for $T = 48(h)$ and $T = 96(h)$, we can calculate the values of $E^{\text{c}}_{\text{max}}$ given $T_c = 2(h)$ or $T_c = 7(h)$, and also the corresponding upper bounds. The results are listed in Table I. The upper bound holds for all cases though the difference between the upper bound and $E^{\text{c}}_{\text{max}}$ increases when T increases. This is due to the fact that for multiple days battery can be repeatedly charged and discharged; however, this fact is not taken into account in the upper bound in Proposition 1.

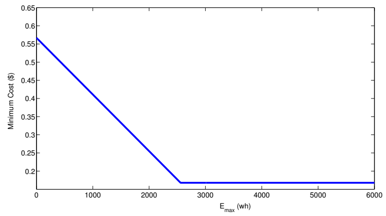
TABLE I

Values and upper bounds on $E^{\text{c}}_{\text{max}}(Wh)$

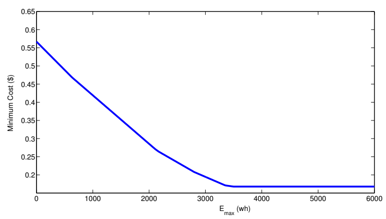
	$T = 24(h)$		$T = 48(h)$		$T = 96(h)$	
$T_c(h)$	Sim	UB	Sim	UB	Sim	UB
2	2560	3095	3100	6051	3100	11742
7	3500	3500	3570	6051	3650	11742

B. Constant Loads

In this subsection, we first set $P_{\text{load}} = 200(W)$, and increase the battery capacity E_{max} from 0 to $1000(Wh)$ with the step size $10(Wh)$. Then for $T_c = 2(h)$ (or $T_c = 7(h)$, respectively), the value for $E^{\text{c}}_{\text{max}}$ is calculated to be $499(Wh)$ (or $700(Wh)$, respectively). If we try to use the result in Proposition 2 by replacing P_{Bmax} and P_{Bmin} in Proposition 2



(a) $T_c = 2(h)$



(b) $T_c = 7(h)$

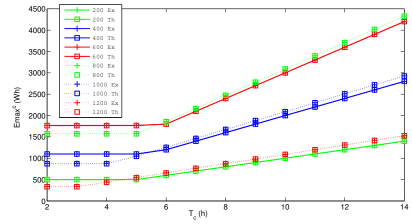
Fig. 4. Plots of minimum costs versus E_{max} obtained via simulations with the load profile in Fig. 3 and $T = 24(h)$.

with Eq. (2) and then solving $E^{\text{c}}_{\text{max}} = \frac{\min(A_1,B_1)}{2}$ as an implicit equation of $E^{\text{c}}_{\text{max}}$, we obtain $E^{\text{c}}_{\text{max}} = 499(Wh)$ for $T_c = 2(h)$, and $E^{\text{c}}_{\text{max}} = 1(Wh)$ for $T_c = 7(h)$. Even though the theoretical value for $T_c = 2(h)$ is very consistent with the value obtained via simulations, the theoretical value for $T_c = 7(h)$ is far from correct. However, if we apply Algorithm 1 to both cases, the theoretical values are $499(Wh)$ and $700(Wh)$, which are consistent with the values obtained via simulations.

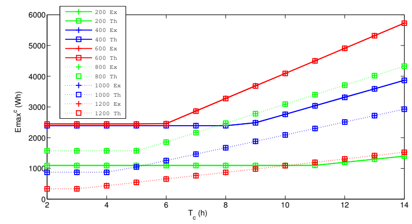
Now we run more simulations to examine the results obtained using Algorithm 1. We choose P_{load} from $200(W)$ to $1200(W)$ with the step size $200(W)$, T_c from $2(h)$ to $14(h)$ with the step size $1(h)$, and T to be $24(h)$, $48(h)$, and $96(h)$, and then calculate $E^{\text{c}}_{\text{max}}$. The results are plotted in Fig. 5. For example, in Fig. 5(a), we fix $T = 24(h)$, and plot the theoretical values (i.e., the curves with the $\square$ marker) and the values obtained via simulations (i.e., the curves with the $+$ marker) of $E^{\text{c}}_{\text{max}}$ as T_c varies from 2 to 14 hours. The theoretical values are very close to the values obtained via simulations for $T = 24(h)$, which is also true for $T = 48(h)$ as shown in Fig. 5(b). However, for $T = 96(h)$, there are slightly larger variations in the difference between the theoretical values and the values obtained via simulations for $P_{\text{load}} = 200(W)$ and $P_{\text{load}} = 400(W)$, as shown in Fig. 5(c). By examining the electricity purchase history (namely, $P_{\text{gp}}(t)$), the discrepancy is due to the fact that the PV generation is not perfectly periodic, which can be verified based on the PV output plot in Fig. 2.

VI. Conclusions

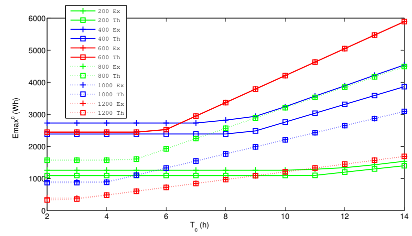
In this paper, we studied optimal sizing of battery storages used in grid-connected PV systems assuming that the maximum battery charging/discharging rate scales linearly with its capacity. In future work, we would like to take into account the dynamic time-of-use pricing of the electricity purchase from the grid, and extend the results to wind energy storage systems.



(a) One day



(b) Two days



(c) Four days

Fig. 5. Plots of the values obtained via simulations (namely, 'Ex') and theoretical values (namely, 'Th') of $E^{\text{c}}_{\text{max}}$ for P_{load} from $200(W)$ to $1200(W)$ with the step size $200(W)$ and $T = 24, 48, 96(h)$.

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