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Optimal Plug-in Electric Vehicle Charging with Schedule Constraints

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Abstract— **This paper proposes a decentralized algorithm that allows a group of Plug-in Electric Vehicles (PEVs) to arrive at an optimal strategy to charge their batteries during the day. By communicating repeatedly with an energy coordinator, the PEVs adjust their battery-charging plans by means of a price-feedback signal that accounts for the aggregated demand. The algorithm allows PEVs to adjust their plan simultaneously while respecting schedule constraints at every iteration. The collective strategy is optimal in that it minimizes the overall price of the supplied energy and leads to an off-peak utilization of the grid. The algorithm is proven to converge to a solution by means of nonlinear analysis tools of discrete-time systems. In order to show convergence, we present a refinement of the LaSalle invariance principle for discrete-time systems. Simulations demonstrate the proficiency of the algorithm in two particular scenarios.**

I. Introduction

Motivation: The large adoption of Plug-in Electric Vehicles (PEV) can bring important economic and environmental benefits as the dependence on petroleum and carbon emissions from the transportation sector would be significantly reduced. However, a large penetration of PEVs would seriously affect the operation of the power grid, from energy production levels to grid capacity, as well as energy prices. Due to this, the development of novel PEV charging strategies which can have a limited impact on the grid is necessary.

Energy pricing models [1], [2] are largely based on the premise that, the larger the power demand is at a given instant of time, the larger the production cost per Kilowatt-hour should be. The increment in the production cost per Kilowatt-hour is not beneficial either for the utility company—which does not increase its profit—or for the end user, who ends up paying a higher rate for the same amount of energy. To lower the burden on both productivity and prices, most approaches on PEV charging advocate for peak-shaving solutions. That is, solutions by means of which PEVs tend to charge in low-demand hours while avoiding to charge in high-demand hours. However, in a more realistic scenario, PEVs may have different usage schedules during the day. In this way, while some vehicles may be able to charge all night long, others may charge during a few hours during the night, and even some vehicles may charge during part of the day. This can lead to situations where vehicles may not be able to charge at exactly the valley hours, which can incur into additional demand peaks, increasing prices for all grid users.

Literature Review: A recent research effort has been articulated to study the implications among the PEV-grid-environment factors. While some works aim to study the impact of penetration of large amounts of PEVs [3], [4], others aim to evaluate the environmental footprint, the potential benefits or drawbacks of different technologies and energy sources [5], [6], [7]. Finally, charge-strategy planning is also receiving increased attention, which is the problem that this work focuses on. The work [1] introduces a decentralized algorithm that computes optimal charging strategies for a large population of PEVs. A bargain is performed between an energy coordinator and the PEVs, which leads to a valley-filling solution that minimizes the overall energy price. In this work, all PEVs are considered to have the same charging schedule. In [8], optimal charging trajectories are computed using linear programming. In particular, two decentralized algorithms to solve the problem are proposed there. The first one requires information about a centralized solution, particularly about the cost function gradient, while the second one assumes that each PEV computes a valley-filling solution based on the average charge requirements from all PEVs. The work in [9] presents an optimization-based strategy that leads to a valley-filling solution. In [10], a centralized PEV charging coordination strategy is proposed in order to shave demand peaks as well as minimize distribution losses. A supervisor controls the battery charging policies for all the PEVs in [11], with the aim of minimizing costs and regulating voltage. Finally, [12] introduces a pricing-based two-layer control algorithm for charging/discharging of PEVs. The algorithm is distributed, exploiting consensus-algorithm ideas. The characterization of the solutions and performance analysis are made via game theory and nonlinear analysis. Neither of the above works considers schedule constraints. In [13], a problem with schedule constraints is considered, using a very similar algorithm to that of [1].

Contributions: The main contribution of this work is a novel Price Leveling algorithm: a decentralized algorithm to compute an optimal charging strategy for each PEV when subject to schedule constraints. We define an optimization problem where the cost function is the overall cost of the energy delivered to the grid, and constraints take into account the daily usage schedule for each vehicle. The algorithm reallocates dynamically charging slots by means of a greedy procedure, that is, vehicles aim to charge at times when energy is cheaper. The allocation does not require the knowledge of the function that determines the price-per Kilowatt due to the instant energy demand. We analyze the algorithm convergence to an optimal solution using invariance theory. To this end, we introduce a novel invariance result, which is an extension of the work in [14], to discrete-time systems.

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Organization: This paper is organized as follows: In Section II, we formulate the PEV charging problem under specified constraints, as an optimization problem, and we also characterize the optimal solution of this problem. In Section III, we introduce the PRICE LEVELING algorithm. Section IV presents the convergence analysis of the introduced algorithm, including some supporting results. In Section V, we show simulation results for a specific scenario. Conclusions and future directions are presented on Section VI.

Notation: In what follows, we let $\mathbb{R}_{\geq 0}$ denote the set of real nonnegative numbers. For a function $f : \mathbb{R} \to \mathbb{R}$, let f' be the derivative of f. Let X be a subset of $\mathbb{R}^n$, and p a point in $\mathbb{R}^n$. We define $\operatorname{dist}(X, p) = \min_{y \in X} \|p - y\|$.

II. OPTIMAL PEV CHARGING WITH SCHEDULE CONSTRAINTS

We consider a set of PEVs, indexed by $i \in I = \{1, \ldots, N\}$, to be charged during the day. The day is discretized into T time slots, indexed by $t \in \tau = \{1, \ldots, T\}$, and each vehicle is able to charge its battery only during the time slots corresponding to a subset $Z_i \subset \tau$, $i \in I$, when it remains idle, either before or after being used. Vehicles aim to utilize a strategy that leads to the minimization of the energy price, as well as the avoidance of peaks of demand on the grid. Each vehicle must be charged according to the following battery dynamics:

$$z_{i,t+1} = z_{i,t} + \frac{\alpha_i}{\beta_i} u_{i,t}, \quad i \in I,$$

where $z_{i,t}$ is the normalized amount of energy in the i^{th} vehicle's battery, at time slot t. Let $z_i \in \mathbb{R}^T_{>0}$ be $z_i = [z_{i,1}, \ldots, z_{i,T}]^\top$, for all $i \in I$. The parameter α_i is the charger efficiency for the vehicle i, and the parameter β_i is the maximum charge capacity of i^{th} vehicle's charge. After a day, it must hold that $z_{i,T} = 1$ for all $i \in I$. Therefore, it must hold that $u_i \in \{u \in \mathbb{R}^T_{\geq 0} \mid u_t = 0, \forall t \notin Z_i, \sum_{t \in \tau} u_t = \gamma_i\}$, where $\gamma_i = (1 - x_{i,1}) \frac{\beta_i}{\alpha_i}$, for $i \in I$. Define the vector $u = [u_1^\top, \ldots, u_N^\top]^\top = [u_{i,t}] \in \mathbb{R}^{NT}$. Then, the control strategy in u aims to minimize the cost function

$$J(u) = \sum_{t \in \tau} p(D_t + \sum_{j \in I} u_{j,t}) \left(D_t + \sum_{j \in I} u_{j,t} \right),$$

where $D \in \mathbb{R}^T_{\geq 0}$ is a known vector that represents the demand on the grid that comes from non-PEV loads, and $p : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ is a continuous function that relates the demand with the electricity price. This cost function corresponds exactly to the overall price of the energy supplied by the power grid [1].

In all, the optimal charging strategies for the PEV correspond to the solution of the following optimization problem:

$$\begin{aligned} &\min_{u \in \mathbb{R}^{NT}_{\geq 0}} J(u), \\ &\text{s.t. } u \in F(\{Z_i\}, \{\gamma_i\}), \end{aligned} \tag{1}$$

where $F(\{Z_i\}, \{\gamma_i\}) = \{u \in \mathbb{R}^{NT}_{\geq 0} \mid u_{i,t} = 0, \ \forall t \notin Z_i, \ \sum_{q \in \tau} u_{i,q} = \gamma_i, \forall i \in I\}$ is the feasible set of the problem.

***Assumption** 2.1 (Properties of the price function p):* The function p is convex, strictly increasing, and there exists $p'_{\min} > 0$, such that $p'(x) \geq p'_{\min}$, for all $x \in F(\{Z_i\}, \{\gamma_i\})$.

Under this assumption, we provide a characterization of the solutions to problem in (1):

***Lemma** 2.1 (Necessary conditions for optimality):* If $u^\star$ is an optimizer of the problem in (1), then:

- for each $i \in I$ and for all $t, q \in Z_i$ such that $u^\star_{i,t}, u^\star_{i,q} > 0$, it holds that $D_t + \sum_j u^\star_{j,t} = D_q + \sum_j u^\star_{j,q}$,
- for each $i \in I$ and for each $q \in Z_i$, such that $u^\star_{i,q} = 0$, it holds that $D_q + \sum_j u^\star_{j,q} \geq D_t + \sum_j u^\star_{j,t}$, for all other $t \in Z_i$ such that $u^\star_{i,t} > 0$. •

Let us introduce some additional shorthand notation. For each $t \in \tau$, we let $x_t = \sum_{i \in I} u_{i,t}$, and given $u^\star$, satisfying the optimal-solution characterization of Lemma 2.1, we let $x^\star_t = \sum_{i \in I} u^\star_{i,t}$. By Lemma 2.1, and given $u^\star$ satisfying the properties of the lemma, we can generate a partition of the set τ, denoted $\{\Upsilon_l\}_{l=1}^{m+1}$, $m < T$, corresponding to the times that result into the same price. In other words, for any pair $t, q \in \Upsilon_l$, $l \in \{1, \ldots, m\}$, it holds that $x^\star_t, x^\star_q > 0$ and $p(D_t + x^\star_t) = p(D_q + x^\star_q)$. The set Υ_{m+1} consists of those $t \in \tau$ such that $x^\star_t = 0$. The collection $\{\Upsilon_l\}_{l=1}^{m+1}$ is ordered according to the corresponding price values, from the cheapest to the priciest. In other words, $l_1 < l_2$, $l_1, l_2 \in \{1, \ldots, m+1\}$, if and only if $p(D_t + x^\star_t) < p(D_q + x^\star_q)$ for all $t \in \Upsilon_{l_1}$, and $q \in \Upsilon_{l_2}$. The following is an immediate consequence of Lemma 2.1.

***Corollary** 2.1 (Optimality leads to best feasible prices):* Let $u^\star$ satisfy the necessary conditions for optimality of Lemma 2.1. If $i \in I$ is such that $u^\star_{i,t} > 0$ for some $t \in \Upsilon_l$, $l \in \{2, \ldots, m\}$, then $Z_i \cap \bigcup_{r=1}^{l-1} \Upsilon_r = \emptyset$. •

Next, for a solution $u^\star$ define the sets $\mathcal{I}_l(u^\star)$, $l \in \{1, \ldots, m\}$ as $\mathcal{I}_l(u^\star) = \{j \in I \mid \exists q \in \Upsilon_l \text{ s.t. } u^\star_{j,q} > 0\}$. With this definition, we have that i) $\mathcal{I}_{l_1}(u^\star) \cap \mathcal{I}_{l_2}(u^\star) = \emptyset$ for all $l_1, l_2 \in \{1, \ldots, m\}$, such that $l_1 \neq l_2$, and ii) $\bigcup_{l=1}^m \mathcal{I}_l(u^\star) = I$. The following result shows that this partition is equal for all optimal solutions $u^\star$ of the problem.

***Lemma** 2.2 (Uniqueness of $x^\star$, $\{\Upsilon_l\}_{l=1}^{m+1}$, and $\{\mathcal{I}_l\}_{l=1}^m$):* Let $u^\star$, and $v^\star$ be optimal solutions of problem in (1) with associated aggregated loads $x^\star_t = \sum_{i \in I} u^\star_{i,t}$ and $y^\star_t = \sum_{i \in I} v^\star_{i,t}$, respectively, and with associated partitions $\{\mathcal{I}_l(u^\star)\}_{l=1}^m$, $\{\mathcal{I}_l(v^\star)\}_{l=1}^m$. Then:

- $x^\star_t = y^\star_t$, for all $t \in \tau$, and $\{\Upsilon_l\}_{l=1}^{m+1}$ is unique,
- $\mathcal{I}_l(u^\star) = \mathcal{I}_l(v^\star) \equiv \mathcal{I}_l$, for all $l \in \{1, \ldots, m\}$. •

III. PRICE LEVELING ALGORITHM

In this section, we propose a PRICE LEVELING algorithm for the PEVs. This is a learning mechanism that the PEVs run at the beginning of the day by interacting repeatedly with the energy coordinator, which is an entity from the utility with the task of determining the demand-based energy price. As in [1], the utility receives the intended battery usage profile from each vehicle and, given the overall demand, i.e., the sum of the non-PEV demand forecast and the PEV population

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demand, it provides feedback by pricing the time slots of the day. Next, according to the new price, the PEVs update their strategy, and send it to the energy coordinator, which again computes the prices. This procedure is run repeatedly until some error-based stop criterion is reached. More precisely, at each iteration, the system evolves following the dynamics $u^{k+1} = g(u^k)$, where $g : \mathbb{R}^{NT}_{\geq 0} \to \mathbb{R}^{NT}_{\geq 0}$ is such that $u^{k+1}_{i,t} = g_{i,t}(u^k)$. This is specified as follows:

$$u^{k+1}_{i,t} = u^k_{i,t} + \sum_{\substack{q \in Z_i \\ p^k_t < p^k_q}} \frac{u^k_{i,q}}{T} \psi_i(p^k_q - p^k_t) - \sum_{\substack{q \in Z_i \\ p^k_t > p^k_q}} \frac{u^k_{i,t}}{T} \psi_i(p^k_t - p^k_q), \tag{2}$$

for each $t \in Z_i$, for all $i \in I$, for each $k \geq 0$. The function $\psi_i : \mathbb{R}_{\geq 0} \to [0, 1]$ is non-decreasing and such that $\psi_i(0) = 0$, for all $i \in I$. We denote $p^k_t = p(D_t + x_t)$, for all $t \in \tau$. Intuitively, each PEV adjusts its charging policy at each iteration with the aim of exploiting the cheapest time slots in its schedule, while avoiding the priciest slots. This learning mechanism can be considered to be a greedy policy, in that it always aims for the cheapest slots. The way the charging strategy for each PEV evolves resembles the structure of a consensus algorithm in that the i^{th} PEV tries to reach consensus in its price for all $t \in Z_i$, manipulating the charge it gets from each time slot. Next, let us introduce some additional shorthand notation. Let us define $L_t = D_t + x_t$, for all $t \in \tau$. Define $L_{\min} = \min_{t \in \tau}(D_t + x_t)$. Besides, given that the amount of charge the set of PEVs is taking during the time window is finite, there exists an upper bound $\overline{L}$ such that $\overline{L} \geq L^k_t$ for all t, k. The following is a sufficient condition that allows us to prove convergence of the algorithm to an optimal profile.

***Assumption** 3.1 (Properties of the functions ψ_i):* Let $x_{\max}$ be such that $x_t \leq x_{\max}$, for all $t \in \tau$. The function ψ_i is Lipschitz, with Lipschitz constant r^i_ψ such that $r^i_\psi < T/((T-1)x_{\max} p'(\overline{L}))$, for all $i \in I$.

The above can be understood as a "coordinating property" of the PEVs' update law, and will be employed as follows. Since p is convex and increasing, we have that $p'(\overline{L}) \geq p'(L^k_t)$, and $p'(L^k_t)$ is the Lipschitz constant of p for the interval $[0, \overline{L}]$. Then, we have that:

$$p(L_1) - p(L_2) \leq p'(\overline{L})(L_1 - L_2),$$

with $L_1 \leq L_2$. Since ψ_i is increasing, and Lipschitz we have:

$$\begin{aligned} \psi(p(L_1) - p(L_2)) &\leq \psi_i(p'(\overline{L})(L_1 - L_2)) \\ &\leq r^i_\psi p'(\overline{L})(L_1 - L_2). \end{aligned}$$

Using Assumption 3.1, with $L_2 = L_{\min}$, we obtain

$$\psi_i(p(L^k_t) - p(L^k_{\min})) < \frac{T}{(T-1)x_{\max}}(L^k_t - L^k_{\min}), \tag{3}$$

for all i, t, k.

***Lemma** 3.1 (Invariance of the feasible set):* The feasible set $F(\{Z_i\}, \{\gamma_i\})$ of the optimization problem in (1) is invariant under the PRICE LEVELING algorithm. •

IV. CONVERGENCE ANALYSIS

In this section, we analyze the convergence of the PRICE LEVELING algorithm introduced in Section III. To this end, we require some results on invariance theory that we present first. These results are a novel discrete-time counterpart of the work for continuous-time systems presented in [14].

A. Invariance Theory

Consider a discrete-time dynamical system given by

$$x^{k+1} = f(x^k), \qquad k \geq 0, \tag{4}$$

where the state x^k belongs to a compact submanifold $\mathcal{M}$ of $\mathbb{R}^n$, and $f : \mathcal{M} \to \mathcal{M}$ is a continuous map. We denote by $\phi(k, x^0)$, $k \geq 0$, a solution starting from the initial condition $x^0 \in \mathcal{M}$. Note that any solution of (4) will be bounded, hence compact.

***Definition** 4.1 (Limit point, Omega-limit set):* Consider a solution of (4), $\phi(\cdot, x^0)$, with initial condition x^0. A point p is said to be a limit point of ϕ if there exists a sequence $\{k_j\}^\infty_{j=0}$, with $k_j \to \infty$ as $j \to \infty$, such that $\lim_{j\to\infty} \phi(k_j, x^0) = p$. The omega-limit set of ϕ denoted as $\Omega(\phi)$ is the set of all limit points of ϕ.

Since $\mathcal{M}$ is compact and f is continuous, the omega-limit set of ϕ is nonempty, closed, invariant under the dynamics in (4), invariantly connected, and it is the smallest set that $\phi(k, x^0)$ approaches as k goes to infinity, see [15], [16].

***Assumption** 4.1 (Height function on $\mathcal{S}$ containing $\Omega(\phi)$):* Assume that:

- $\Omega(\phi)$ is contained in a submanifold $\mathcal{S} \subseteq \mathcal{M}$.
- There exists a compact neighborhood K of $\Omega(\phi)$ in $\mathcal{M}$, such that $O = \text{int}(K)$ is an open neighborhood of $\Omega(\phi)$.
- There is a continuous function $W : K \to \mathbb{R}$ such that $W(f(x)) - W(x) \leq 0$ on $\mathcal{S} \cap O$. Let E be the defined as $E = \{x \in S \cap O \mid W(f(x)) - W(x) = 0\}$. Then $W(f(x)) - W(x) < 0$ on $(\mathcal{S} \cap O) \setminus E$.

The following results are inspired by [14]. The invariance Lemma 4.2 admits a generalization to E being contained in a countable number of level sets of W as in [14], but we state a version that is sufficient to prove our main result.

***Lemma** 4.1:* Let Assumption 4.1, on the existence of a height function on $\mathcal{S}$ containing $\Omega(\phi)$, hold. Then, it must be that $\Omega(\phi) \cap E \neq \emptyset$. •

***Lemma** 4.2 (Invariance Result):* Let Assumption 4.1, on the existence of a height function on $\mathcal{S}$ containing $\Omega(\phi)$ hold. If E is contained in a single level set of W, then $\Omega(\phi) \subseteq E$.

Proof: This proof follows the proof of the first and second cases on Theorem 4 in [14]. The difference lies in that a solution ϕ of a discrete-time system is discontinuous, then it is necessary to show that if the solution approaches arbitrarily a limit point p where $W \circ f - W$ is zero, then it is a contradiction that it can approach another point q for which $W \circ f - W$ is negative. Extended details will appear in a forthcoming publication. ■

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B. Main Result

***Theorem** 4.1:* The PRICE LEVELING algorithm defined by equation (2) converges to an optimal solution of the problem.

Proof: Let us define functions V_l, $l \in \{1, \ldots, m\}$, with $m \leq T$, where

$$V_l(u) = \min_{q \in \Theta_l} p(D_q + x_q^\star) - \min_{t \in \Theta_l} p(D_t + x_t), \tag{5}$$

$$\Theta_l = \cup_{r=l}^{m+1} \Upsilon_r. \tag{6}$$

Recall that $x_t = \sum_{i \in I} u_{i,t}$. Define sets $\mathcal{E}_l$, for $l \in \{0, \ldots, m\}$, such that $\mathcal{E}_0 = F(\{Z_i\}, \{\gamma_i\})$ and

$$\begin{aligned}\mathcal{E}_l =&\{u \in \mathcal{E}_{l-1} \mid x_t = x_t^\star, \forall t \in \bigcup_{r=1}^{l} \Upsilon_r, \\ &D_q + x_q \geq D_t + x_t, \forall t \in \bigcup_{r=1}^{l} \Upsilon_r, q \in \tau \setminus \bigcup_{r=1}^{l} \Upsilon_r\},\end{aligned} \tag{7}$$

$l \in \{1, \ldots, m\}$. Note that, if $u \in \mathcal{E}_l$, then $u_{i,t}$, with $i \in I$, and $t \in \cup_{r=1}^{l} \Upsilon_r$ can be extended to an optimal solution profile $v_{i,t}$, $i \in I$, $t \in \tau$ as follows:

$$v_{i,t} = \begin{cases} u_{i,t}, & i \in I, \text{ and } t \in \tau \setminus \Theta_{l+1} = \cup_{r=1}^{l} \Upsilon_r, \\ u_{i,t}^\star, & i \in I, \text{ and } t \in \Theta_{l+1} = \cup_{r=l+1}^{m+1} \Upsilon_r. \end{cases}$$

Then v defined above satisfies the necessary conditions of optimality in Lemma 2.1. The proof is based on showing that V_l is decreasing in $\mathcal{E}_{l-1} \setminus \mathcal{E}_l$, while $V_l = 0$, for each $l \in \{1, \ldots, m\}$. Then, we use it, along with Lemma 4.2 to systematically conclude that the Omega-limit set of a solution lies in $\mathcal{E}_l$, for each $l \in \{1, \ldots, m\}$ until $\mathcal{E}_m$ corresponds to the optimizer set of the problem in (1). Full details will be shown in a forthcoming publication. ■

V. SIMULATIONS

In order to demonstrate the PRICE LEVELING algorithm performance, we carry out simulations for two different cases. For both cases, we have a scenario of 24 hours, starting at 12:00 p.m. and ending at the same time the next day. This time window is divided in 48 time slots with equal duration, that is, each slot is half-hour long. For both cases we consider the same non-PEV demand profile, and a population of 20 PEVs. Each PEV has a particular energy requirement, given by γ_i for $i \in \{1, \ldots, 20\}$. We consider a function p such that $p(x) = x^2$. The functions ψ_i are linear functions of the form $\psi_i(x) = r_\psi^i x$, with $r_\psi^i = r_\psi$ for all $i \in I$. Even though our theoretical result presents an upper bound on the Lipschitz constant for the functions ψ_i, we use constants larger than the bound, which still lead to convergence in smaller time. If we use Lipschitz constants r_ψ^i that satisfy the theoretical bounds, the algorithm may take as much as 8000 iterations to converge. This number of iterations may vary depending on the initial conditions and the parameters D, γ_i, Z_i for all $i \in I$.

For the first case, we have that the first PEV can charge from 20:00 to 1:00. The second can charge from 20:30 to 1:30. Each PEV i can charge starting half-hour after $i-1$ starts, during exactly 5 hours, thus, the 20$^{\text{th}}$ PEV can charge from 5:30 to 10:30.

For the second case, we have that out of the 20 PEVs, 6 can charge during the whole time window, and 14 PEVs can charge from 12:00 to 2:30 and from 5:00 to 12:00. Then, each element of the second group of PEVs has a disconnected Z_i.

On Figures 1, 4 we show the non-PEV demand profile, colored in light grey, while the PEV demand is shown in dark colors. In both cases, each PEV has as an initial charging strategy to obtain the same amount of energy from each $q \in Z_i$. On Figure 2,we show that the different Z_i and γ_i for all $i \in I$, joint with the values of D lead to a two-level optimal solution. It means that PEVs will be only divided in two groups of different prices. On Figure 5, a two-level optimal configuration is achieved. Note that the higher level is split into two groups of time slots. This is due to the choice of

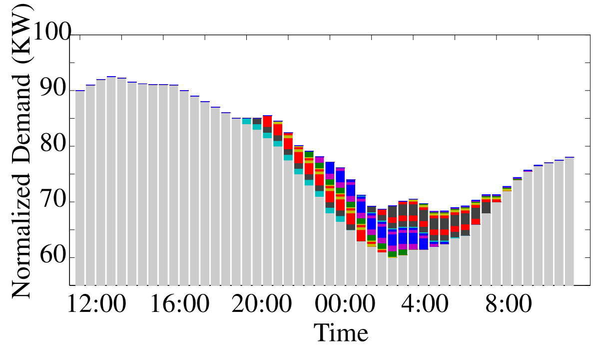

Fig. 1. Initial demand profile. The non-PEV demand is shown in light grey. Other colors show each PEV's charging profile.

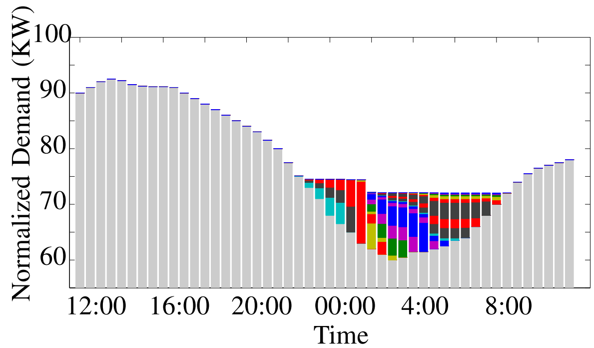

Fig. 2. Optimal demand profile. The non-PEV demand is shown in light grey. Other colors show each PEV's charging profile.

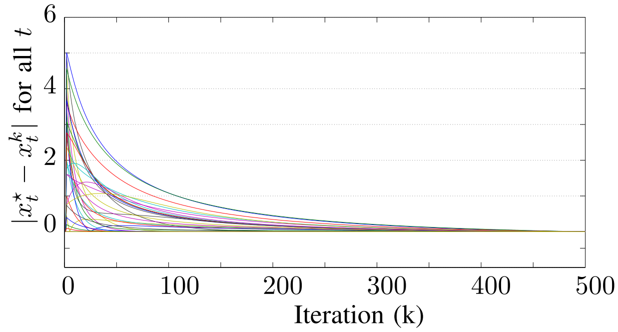

Fig. 3. Error on x_t for each $t \in \tau$.

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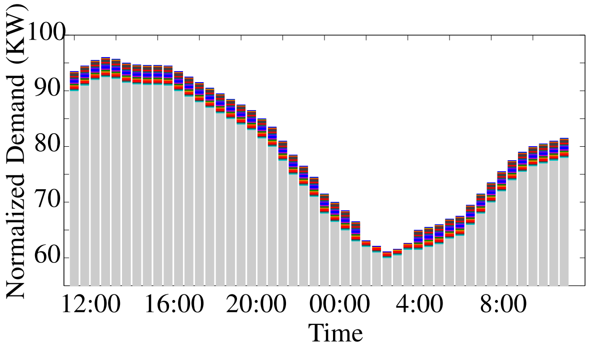

Fig. 4. Initial demand profile. The non-PEV demand is shown in light grey. Other colors show each PEV's charging profile.

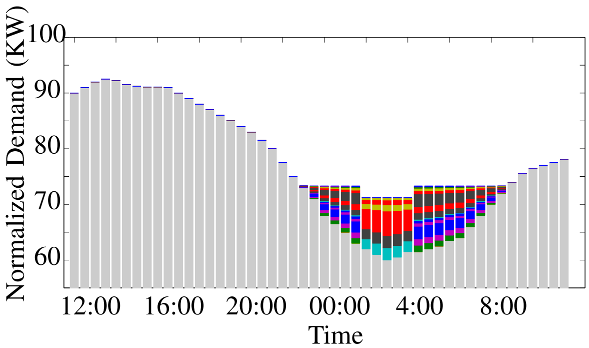

Fig. 5. Optimal demand profile. The non-PEV demand is shown in light grey. Other colors show each PEV's charging profile.

Z_i for the 14 PEVs that cannot charge in the lowest-demand slots. Figures 3, 6 show the convergence of the x_t^k towards $x_t^\star$ for each t as iterations go by.

VI. Conclusions and Future Work

This work addresses a problem of computing charging strategies for a group of PEV subject to schedule constraints, which can optimize the overall energy cost supplied by the grid. A complete characterization of the solution set for this problem is provided. We propose a decentralized PRICE LEVELING algorithm that satisfies the constraints at any iteration. The algorithm does not require PEVs to know the pricing function, instead it is based on a learning approach which does not require a direct gradient estimation. To this end, an invariance result for discrete-time systems is presented that allows us to provide convergence guarantees towards an optimal solution. Finally, an application example with simulation is shown, in order to demonstrate the algorithm performance.

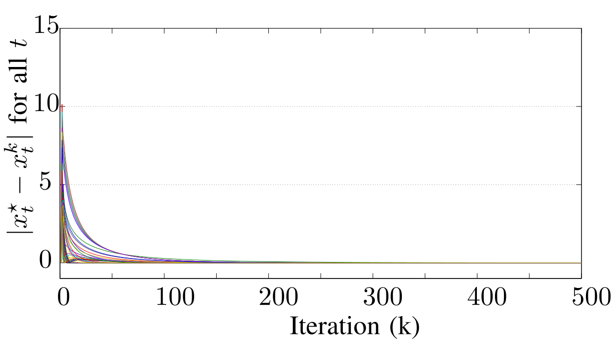

Fig. 6. Error on x_t for each $t \in \tau$.

In future work, we will aim to study how to include additional constraints the usage schedule to account for how much energy each vehicle may spend during certain hours. The convergence speed as well as the algorithm convergence properties under time-varying interactions between PEVs and the energy coordinator are being investigated as well.

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