

Storage Size Determination for Grid-Connected Photovoltaic Systems

Yu Ru, Jan Kleissl, and Sonia Martinez

Abstract—In this paper, we study the problem of determining the size of battery storage used in grid-connected photovoltaic (PV) systems. In our setting, electricity is generated from PV and is used to supply the demand from loads. Excess electricity generated from the PV can be either sold back to the grid or stored in a battery, and electricity must be purchased from the electric grid if the PV generation and battery discharging cannot meet the demand. Due to the time-of-use electricity pricing and net metered PV systems, electricity can also be purchased from the grid when the price is low, and be sold back to the grid when the price is high. The objective is to minimize the cost associated with net power purchase from the electric grid and the battery capacity loss while at the same time satisfying the load and reducing the peak electricity purchase from the grid. Essentially, the objective function depends on the chosen battery size. We want to find a unique critical value (denoted as C_{ref}^c) of the battery size such that the total cost remains the same if the battery size is larger than or equal to C_{ref}^c , and the cost is strictly larger if the battery size is smaller than C_{ref}^c . We obtain a criterion for evaluating the economic value of batteries compared to purchasing electricity from the grid, propose lower and upper bounds on C_{ref}^c , and introduce an efficient algorithm for calculating its value; these results are validated via simulations.

Index Terms—PV, Grid, Battery, Optimization

I. INTRODUCTION

The need to reduce greenhouse gas emissions due to fossil fuels and the liberalization of the electricity market have led to large scale development of renewable energy generators in electric grids [1]. Among renewable energy technologies such as hydroelectric, photovoltaic (PV), wind, geothermal, biomass, and tidal systems, grid-connected solar PV continued to be the fastest growing power generation technology, with a 70% increase in existing capacity to 13GW in 2008 [2]. However, solar energy generation tends to be variable due to the diurnal cycle of the solar geometry and clouds. Storage devices (such as batteries, ultracapacitors, compressed air, and pumped hydro storage [3]) can be used to i) smooth out the fluctuation of the PV output fed into electric grids (“capacity firming”) [2], [4], ii) discharge and augment the PV output during times of peak energy usage (“peak shaving”) [5], iii) store energy for nighttime use, for example in zero-energy buildings and residential homes, or iv) power arbitrage under time-of-use electricity pricing, i.e., storing surplus PV when the time-of-use price is low and selling when the time-of-use price is high.

Yu Ru, Jan Kleissl, and Sonia Martinez are with the Mechanical and Aerospace Engineering Department, University of California, San Diego (e-mail: yuru2@ucsd.edu, jkleissl@ucsd.edu, soniamd@ucsd.edu).

Depending on the specific application (whether it is off-grid or grid-connected), battery storage size is determined based on the battery specifications such as the battery storage capacity (and the minimum battery charging/discharging time). For off-grid applications, batteries have to fulfill the following requirements: (i) the maximum discharging rate has to be larger than or equal to the peak load capacity; (ii) the battery storage capacity has to be large enough to supply the largest night time energy use and to be able to supply energy during the longest cloudy period (autonomy). The IEEE standard [6] provides sizing recommendations for lead-acid batteries in stand-alone PV systems. In [7], the solar panel size and the battery size have been selected via simulations to optimize the operation of a stand-alone PV system, which considers reliability measures in terms of loss of load hours, the energy loss and the total cost. In contrast, if the PV system is grid-connected, autonomy is a secondary goal; instead, batteries can reduce the fluctuation of PV output or provide economic benefits such as demand charge reduction, and power arbitrage. The work in [8] analyzes the relation between available battery capacity and output smoothing, and estimates the required battery capacity using simulations. In addition, the battery sizing problem has been studied for wind power applications [9]–[11] and hybrid wind/solar power applications [12]–[14]. In [9], design of a battery energy storage system is examined for the purpose of attenuating the effects of unsteady input power from wind farms, and solution to the problem via a computational procedure results in the determination of the battery energy storage system’s capacity. Similarly, in [11], based on the statistics of long-term wind speed data captured at the farm, a dispatch strategy is proposed which allows the battery capacity to be determined so as to maximize a defined service lifetime/unit cost index of the energy storage system; then a numerical approach is used due to the lack of an explicit mathematical expression to describe the lifetime as a function of the battery capacity. In [10], sizing and control methodologies for a zinc-bromine flow battery based energy storage system are proposed to minimize the cost of the energy storage system. However, the sizing of the battery is significantly impacted by specific control strategies. In [12], a methodology for calculating the optimum size of a battery bank and the PV array for a stand-alone hybrid wind/PV system is developed, and a simulation model is used to examine different combinations of the number of PV modules and the number of batteries. In [13], an approach is proposed to help designers determine the optimal design of a hybrid wind-solar power system; the proposed analysis employs linear programming techniques to minimize the average production cost of electricity while meeting the load requirements in

a reliable manner. In [14], genetic algorithms are used to optimally size the hybrid system components, i.e., to select the optimal wind turbine and PV rated power, battery energy storage system nominal capacity, and inverter rating. The primary objective is the minimization of the levelized cost of energy of the island system.

In this paper, we study the problem of determining the battery size for grid-connected PV systems for the purpose of power arbitrage and peak shaving. The targeted applications are primarily electricity customers with PV arrays “behind the meter,” such as most residential and commercial buildings with rooftop PVs. In such applications, the objective is to minimize the investment on battery storage (or equivalently, the battery size if the battery technology is chosen) while minimizing the net power purchase from the grid subject to the load, battery aging, and peak power reduction (instead of smoothing out the fluctuation of the PV output fed into electric grids). Our setting¹ is shown in Fig. 1. We assume that the grid-connected PV system utilizes time-of-use pricing (i.e., electricity prices change according to a fixed daily schedule) and that the prices for sales and purchases of electricity at any time are identical. In other words, the grid-connected PV system is net metered, a policy which has been widely adopted in most states in US [15] and some other countries in Europe, Australia, and North America. Electricity is generated from PV panels, and is used to supply different types of loads. Battery storage is used to either store excess electricity generated from PV systems when the time-of-use price is lower for selling back to the grid when the time-of-use price is higher (this is explained in detail in Section V-B), or purchase electricity from the grid when the time-of-use pricing is lower and sell back to the grid when the time-of-use pricing is higher (power arbitrage). Without a battery, if the load was too large to be supplied by PV generated electricity, electricity would have to be purchased from the grid to meet the demand. Naturally, given the high cost of battery storage, the size of the battery storage should be chosen such that the cost of electricity purchase from the grid and the loss due to battery aging are minimized. Intuitively, if the battery is too large, the electricity purchase cost could be the same as the case with a relatively smaller battery. In this paper, we show that there is a unique critical value (denoted as C_{ref}^c , refer to Problem 1) of the battery capacity such that the cost of electricity purchase and the loss due to battery aging remains the same if the battery size is larger than or equal to C_{ref}^c , and the cost is strictly larger if the battery size is smaller than C_{ref}^c . We obtain a criterion for evaluating the economic value of batteries compared to purchasing electricity from the grid, propose lower and upper bounds on C_{ref}^c given the PV generation, loads, and the time period for minimizing the costs, and introduce an efficient algorithm for calculating the critical battery capacity based on the bounds; these results are validated via simulations.

The contributions of this work are the following: i) to the best of our knowledge, this is the first attempt on determining

¹Note that solar panels and batteries both operate on DC, while the grid and loads operate on AC. Therefore, DC-to-AC and AC-to-DC power conversion is necessary when connecting solar panels and batteries with the grid and loads.

lower and upper bounds of the battery size for grid-connected, net metered PV systems based on a theoretical analysis; in contrast, most previous work are based on trial and error approaches, e.g., the work in [8]–[12] (one exception is the theoretical work in [16], in which the goal is to minimize the long-term average electricity costs in the presence of dynamic pricing as well as investment in storage in a stochastic setting; however, no battery aging is taken into account and special pricing structure is assumed); ii) a criterion for evaluating the economic value of batteries compared to purchasing electricity from the grid is derived (refer to Proposition 4 and Assumption 2), which can be easily calculated and could be potentially used for choosing appropriate battery technologies for practical applications; and iii) lower and upper bounds on the battery size are proposed, and an efficient algorithm is introduced to calculate its value for the given PV generation and dynamic loads; these results are then validated using simulations. Simulation results illustrate the benefits of employing batteries in grid-connected PV systems via peak shaving and cost reductions compared with the case without batteries (this is discussed in Section V-B).

The paper is organized as follows. In the next section, we lay out our setting, and formulate the storage size determination problem. Lower and upper bounds on C_{ref}^c are proposed in Section III. Algorithms are introduced in Section IV to calculate the value of the critical battery capacity. In Section V, we validate the results via simulations. Finally, conclusions and future directions are given in Section VI.

II. PROBLEM FORMULATION

In this section, we formulate the problem of determining the storage size for a grid-connected PV system, as shown in Fig. 1. We first introduce different components in our setting.

A. Photovoltaic Generation

We use the following equation to calculate the electricity generated from solar panels:

$$P_{\text{pv}}(t) = \text{GHI}(t) \times S \times \eta, \quad (1)$$

where

- $\text{GHI} (Wm^{-2})$ is the global horizontal irradiation at the location of solar panels,

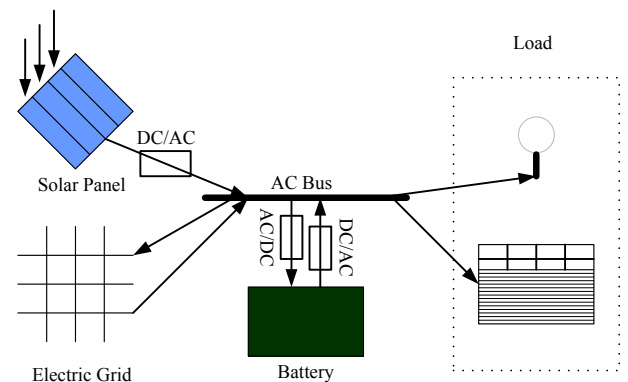


Fig. 1. Grid-connected PV system with battery storage and loads.

- S (m^2) is the total area of solar panels, and
- η is the solar conversion efficiency of the PV cells.

The PV generation model is a simplified version of the one used in [17] and does not account for PV panel temperature effects.²

B. Electric Grid

Electricity can be purchased from or sold back to the grid. For simplicity, we assume that the prices for sales and purchases at time t are identical and are denoted as $C_g(t)$ ($\$/Wh$). Time-of-use pricing is used ($C_g(t) \geq 0$ depends on t) because commercial buildings with PV systems that would consider a battery system usually pay time-of-use electricity rates. In addition, with increased deployment of smart meters and electric vehicles, some utility companies are moving towards residential time-of-use pricing as well; for example, SDG&E (San Diego Gas & Electric) has the peak, semipeak, offpeak prices for a day in the summer season [18], as shown in Fig. 3.

We use $P_g(t)(W)$ to denote the electric power exchanged with the grid with the interpretation that

- $P_g(t) > 0$ if electric power is purchased from the grid,
- $P_g(t) < 0$ if electric power is sold back to the grid.

In this way, positive costs are associated with the electricity purchase from the grid, and negative costs with the electricity sold back to the grid. In this paper, peak shaving is enforced through the constraint that

$$P_g(t) \leq D ,$$

where D is a positive constant.

C. Battery

A battery has the following dynamic:

$$\frac{dE_B(t)}{dt} = P_B(t) , \quad (2)$$

where $E_B(t)(Wh)$ is the amount of electricity stored in the battery at time t , and $P_B(t)(W)$ is the charging/discharging rate; more specifically,

- $P_B(t) > 0$ if the battery is charging,
- and $P_B(t) < 0$ if the battery is discharging.

For certain types of batteries, higher order models exist (e.g., a third order model is proposed in [19], [20]).

To take into account battery aging, we use $C(t)(Wh)$ to denote the usable battery capacity at time t . At the initial time t_0 , the usable battery capacity is C_{ref} , i.e., $C(t_0) = C_{ref} \geq 0$. The cumulative capacity loss at time t is denoted as $\Delta C(t)(Wh)$, and $\Delta C(t_0) = 0$. Therefore,

$$C(t) = C_{ref} - \Delta C(t) .$$

The battery aging satisfies the following dynamic equation

$$\frac{d\Delta C(t)}{dt} = \begin{cases} -ZP_B(t) & \text{if } P_B(t) < 0 \\ 0 & \text{otherwise} , \end{cases} \quad (3)$$

²Note that our analysis on determining battery capacity only relies on $P_{pv}(t)$ instead of detailed models of the PV generation. Therefore, more complicated PV generation models can also be incorporated into the cost minimization problem as discussed in Section II.F.

where $Z > 0$ is a constant depending on battery technologies. This aging model is derived from the aging model in [5] under certain reasonable assumptions; the detailed derivation is provided in the Appendix. Note that there is a capacity loss only when electricity is discharged from the battery. Therefore, $\Delta C(t)$ is a nonnegative and non-decreasing function of t .

We consider the following constraints on the battery:

- At any time, the battery charge $E_B(t)$ should satisfy

$$0 \leq E_B(t) \leq C(t) = C_{ref} - \Delta C(t) ,$$

- The battery charging/discharging rate should satisfy

$$P_{Bmin} \leq P_B(t) \leq P_{Bmax} ,$$

where $P_{Bmin} < 0$, $-P_{Bmin}$ is the maximum battery discharging rate, and $P_{Bmax} > 0$ is the maximum battery charging rate. For simplicity, we assume that

$$P_{Bmax} = -P_{Bmin} = \frac{C(t)}{T_c} = \frac{C_{ref} - \Delta C(t)}{T_c} ,$$

where constant $T_c > 0$ is the minimum time required to charge the battery from 0 to $C(t)$ or discharge the battery from $C(t)$ to 0.

D. Load

$P_{load}(t)(W)$ denotes the load at time t . We do not make explicit assumptions on the load considered in Section III except that $P_{load}(t)$ is a (piecewise) continuous function. In residential home settings, loads could have a fixed schedule such as lights and TVs, or a relatively flexible schedule such as refrigerators and air conditioners. For example, air conditioners can be turned on and off with different schedules as long as the room temperature is within a comfortable range.

E. Converters for PV and Battery

Note that the PV, battery, grid, and loads are all connected to an AC bus. Since PV generation is operated on DC, a DC-to-AC converter is necessary, and its efficiency is assumed to be a constant η_{pv} satisfying

$$0 < \eta_{pv} \leq 1 .$$

Since the battery is also operated on DC, an AC-to-DC converter is necessary when charging the battery, and a DC-to-AC converter is necessary when discharging as shown in Fig. 1. For simplicity, we assume that both converters have the same constant conversion efficiency η_B satisfying

$$0 < \eta_B \leq 1 .$$

We define

$$P_{BC}(t) = \begin{cases} \eta_B P_B(t) & \text{if } P_B(t) < 0 \\ \frac{P_B(t)}{\eta_B} & \text{otherwise} , \end{cases}$$

In other words, $P_{BC}(t)$ is the power exchanged with the AC bus when the converters and the battery are treated as an entity. Similarly, we can derive

$$P_B(t) = \begin{cases} \frac{P_{BC}(t)}{\eta_B} & \text{if } P_{BC}(t) < 0 \\ \eta_B P_{BC}(t) & \text{otherwise} . \end{cases}$$

Note that η_B^2 is the round trip efficiency of the battery converters.

F. Cost Minimization

With all the components introduced earlier, now we can formulate the following problem of minimizing the sum of the net power purchase cost³ and the cost associated with the battery capacity loss while guaranteeing that the demand from loads and the peak shaving requirement are satisfied:

$$\begin{aligned}
& \min_{P_B, P_g} \int_{t_0}^{t_0+T} C_g(\tau) P_g(\tau) d\tau + K \Delta C(t_0 + T) \\
& \text{s.t. } \eta_{pv} P_{pv}(t) + P_g(t) = P_{BC}(t) + P_{load}(t), \quad (6) \\
& \frac{dE_B(t)}{dt} = P_B(t), \\
& \frac{d\Delta C(t)}{dt} = \begin{cases} -Z P_B(t) & \text{if } P_B(t) < 0 \\ 0 & \text{otherwise,} \end{cases} \\
& 0 \leq E_B(t) \leq C_{ref} - \Delta C(t), \\
& E_B(t_0) = 0, \Delta C(t_0) = 0, \\
& P_{Bmin} \leq P_B(t) \leq P_{Bmax}, \\
& P_{Bmax} = -P_{Bmin} = \frac{C_{ref} - \Delta C(t)}{T_c}, \\
& P_B(t) = \begin{cases} \frac{P_{BC}(t)}{\eta_B} & \text{if } P_{BC}(t) < 0 \\ \eta_B P_{BC}(t) & \text{otherwise,} \end{cases} \\
& P_g(t) \leq D, \quad (7)
\end{aligned}$$

where t_0 is the initial time, T is the time period considered for the cost minimization, $K(\$/Wh) > 0$ is the unit cost for the battery capacity loss. Note that:

- i) No cost is associated with PV generation. In other words, PV generated electricity is assumed free;
- ii) $K\Delta C(t_0 + T)$ is the loss of the battery purchase investment during the time period from t_0 to $t_0 + T$ due to the use of the battery to reduce the net power purchase cost;
- iii) Eq. (6) is the power balance requirement for any time $t \in [t_0, t_0 + T]$;
- iv) the constraint $P_g(t) \leq D$ captures the peak shaving requirement.

Given a battery of initial capacity C_{ref} , on the one hand, if the battery is rarely used, then the cost due to the capacity loss $K\Delta C(t_0 + T)$ is low while the net power purchase cost $\int_{t_0}^{t_0+T} C_g(\tau) P_g(\tau) d\tau$ is high; on the other hand, if the battery is used very often, then the net power purchase cost $\int_{t_0}^{t_0+T} C_g(\tau) P_g(\tau) d\tau$ is low while the cost due to the capacity loss $K\Delta C(t_0 + T)$ is high. Therefore, there is a tradeoff on the use of the battery, which is characterized by calculating an optimal control policy on $P_B(t), P_g(t)$ to the optimization problem in Eq. (7).

Remark 1 Besides the constraint $P_g(t) \leq D$, peak shaving is also accomplished indirectly through dynamic pricing. Time-of-use price margins and schedules are motivated by the peak load magnitude and timing. Minimizing the net power purchase cost results in battery discharge and reduction in grid purchase during peak times. If, however, the peak load for a

³Note that the net power purchase cost include the positive cost to purchase electricity from the grid, and the negative cost to sell electricity back to the grid.

customer falls into the off-peak time period, then the constraint on $P_g(t)$ limits the amount of electricity that can be purchased. Peak shaving capabilities of the constraint $P_g(t) \leq D$ and dynamic pricing will be illustrated in Section V-B. ■

G. Storage Size Determination

Based on Eq. (6), we obtain

$$P_g(t) = P_{load}(t) - \eta_{pv} P_{pv}(t) + P_{BC}(t).$$

Let $u(t) = P_{BC}(t)$, then the optimization problem in Eq. (7) can be rewritten as

$$\begin{aligned}
& \min_u \int_{t_0}^{t_0+T} C_g(\tau) (P_{load}(\tau) - \eta_{pv} P_{pv}(\tau) + u(\tau)) d\tau + K \Delta C(t_0 + T) \\
& \text{s.t. } \frac{dE_B(t)}{dt} = \begin{cases} \frac{u(t)}{\eta_B} & \text{if } u(t) < 0 \\ \eta_B u(t) & \text{otherwise,} \end{cases} \\
& \frac{d\Delta C(t)}{dt} = \begin{cases} -Z \frac{u(t)}{\eta_B} & \text{if } u(t) < 0 \\ 0 & \text{otherwise,} \end{cases} \\
& E_B(t) \geq 0, E_B(t_0) = 0, \Delta C(t_0) = 0, \\
& E_B(t) + \Delta C(t) \leq C_{ref}, \\
& \eta_B u(t) T_c + \Delta C(t) \leq C_{ref} \text{ if } u(t) > 0, \\
& -\frac{u(t)}{\eta_B} T_c + \Delta C(t) \leq C_{ref} \text{ if } u(t) < 0, \\
& P_{load}(t) - \eta_{pv} P_{pv}(t) + u(t) \leq D. \quad (8)
\end{aligned}$$

Now it is clear that only $u(t)$ is an independent variable. We define the set of feasible controls as controls that guarantee all the constraints in the optimization problem in Eq. (8).

Let J denote the objective function

$$\min_u \int_{t_0}^{t_0+T} C_g(\tau) (P_{load}(\tau) - \eta_{pv} P_{pv}(\tau) + u(\tau)) d\tau + K \Delta C(t_0 + T).$$

If we fix the parameters t_0, T, K, Z, T_c , and D , J is a function of C_{ref} , which is denoted as $J(C_{ref})$. If we increase C_{ref} , intuitively J will decrease though may not strictly decrease (this is formally proved in Proposition 2) because the battery can be utilized to decrease the cost by

- i) storing extra electricity generated from PV or purchasing electricity from the grid when the time-of-use pricing is low, and
- ii) supplying the load or selling back when the time-of-use pricing is high.

Now we formulate the following storage size determination problem.

Problem 1 (Storage Size Determination) Given the optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c , and D , determine a critical value $C_{ref}^c \geq 0$ such that

- $\forall C_{ref} < C_{ref}^c, J(C_{ref}) > J(C_{ref}^c)$, and
- $\forall C_{ref} \geq C_{ref}^c, J(C_{ref}) = J(C_{ref}^c)$.

One approach to calculate the critical value C_{ref}^c is that we first obtain an explicit expression for the function $J(C_{ref})$ by solving the optimization problem in Eq. (8) and then solve for C_{ref}^c based on the function J . However, the optimization problem in Eq. (8) is difficult to solve due to the nonlinear constraints on $u(t)$ and $E_B(t)$, and the fact that it is hard to

obtain analytical expressions for $P_{\text{load}}(t)$ and $P_{\text{pv}}(t)$ in reality. Even though it might be possible to find the optimal control using the minimum principle [21], it is still hard to get an explicit expression for the cost function J . Instead, in the next section, we identify conditions under which the storage size determination problem results in non-trivial solutions (namely, C_{ref}^c is positive and finite), and then propose lower and upper bounds on the critical battery capacity C_{ref}^c .

III. BOUNDS ON C_{ref}^c

Now we examine the cost minimization problem in Eq. (8). Since $P_{\text{load}}(t) - \eta_{\text{pv}}P_{\text{pv}}(t) + u(t) \leq D$, or equivalently, $u(t) \leq D + \eta_{\text{pv}}P_{\text{pv}}(t) - P_{\text{load}}(t)$, is a constraint that has to be satisfied for any $t \in [t_0, t_0 + T]$, there are scenarios in which either there is no feasible control or $u(t) = 0$ for $t \in [t_0, t_0 + T]$. Given $P_{\text{pv}}(t)$, $P_{\text{load}}(t)$, and D , we define

$$S_1 = \{t \in [t_0, t_0 + T] \mid D + \eta_{\text{pv}}P_{\text{pv}}(t) - P_{\text{load}}(t) < 0\}, \quad (9)$$

$$S_2 = \{t \in [t_0, t_0 + T] \mid D + \eta_{\text{pv}}P_{\text{pv}}(t) - P_{\text{load}}(t) = 0\}, \quad (10)$$

$$S_3 = \{t \in [t_0, t_0 + T] \mid D + \eta_{\text{pv}}P_{\text{pv}}(t) - P_{\text{load}}(t) > 0\}. \quad (11)$$

Note that⁴ $S_1 \oplus S_2 \oplus S_3 = [t_0, t_0 + T]$. Intuitively, S_1 is the set of time instants at which the battery can only be discharged, S_2 is the set of time instants at which the battery can be discharged or is not used (i.e., $u(t) = 0$), and S_3 is the set of time instants at which the battery can be charged, discharged, or is not used.

Proposition 1 Given the optimization problem in Eq. (8), if

- i) $t_0 \in S_1$, or
- ii) S_3 is empty, or
- iii) $t_0 \notin S_1$ (or equivalently, $t_0 \in S_2 \cup S_3$), S_3 is nonempty, S_1 is nonempty, and $\exists t_1 \in S_1, \forall t_3 \in S_3, t_1 < t_3$,

then either there is no feasible control or $u(t) = 0$ for $t \in [t_0, t_0 + T]$.

Proof: Now we prove that, under these three cases, either there is no feasible control or $u(t) = 0$ for $t \in [t_0, t_0 + T]$.

- i) If $t_0 \in S_1$, then $u(t_0) \leq D + \eta_{\text{pv}}P_{\text{pv}}(t_0) - P_{\text{load}}(t_0) < 0$. However, since $E_B(t_0) = 0$, the battery cannot be discharged at time t_0 . Therefore, there is no feasible control.
- ii) If S_3 is empty, it means that $u(t_0) \leq D + \eta_{\text{pv}}P_{\text{pv}}(t_0) - P_{\text{load}}(t_0) \leq 0$ for any $t \in [t_0, t_0 + T]$, which implies that the battery can never be charged. If S_1 is nonempty, then there exists some time instant when the battery has to be discharged. Since $E_B(t_0) = 0$ and the battery can never be charged, there is no feasible control. If S_1 is empty, then $u(t) = 0$ for any $t \in [t_0, t_0 + T]$ is the only feasible control because $E_B(t_0) = 0$.
- iii) In this case, the battery has to be discharged at time t_1 , but the charging can only happen at time instant $t_3 \in S_3$. If $\forall t_3 \in S_3, \exists t_1 \in S_1$ such that $t_1 < t_3$, then the battery is always discharged before possibly being charged. Since $E_B(t_0) = 0$, then there is no feasible control. ■

⁴ $C = A \oplus B$ means $C = A \cup B$ and $A \cap B = \emptyset$.

Note that if the only feasible control is $u(t) = 0$ for $t \in [t_0, t_0 + T]$, then the battery is not used. Therefore, we impose the following assumption.

Assumption 1 In the optimization problem in Eq. (8), $t_0 \in S_2 \cup S_3$, S_3 is nonempty, and either

- S_1 is empty, or
- S_1 is nonempty, but $\forall t_1 \in S_1, \exists t_3 \in S_3, t_3 < t_1$,

where S_1, S_2, S_3 are defined in Eqs. (9), (10), (11).

Given Assumption 1, there exists at least one feasible control. Now we examine how $J(C_{\text{ref}})$ changes when C_{ref} increases.

Proposition 2 Consider the optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c , and D . If $C_{\text{ref}}^1 < C_{\text{ref}}^2$, then $J(C_{\text{ref}}^1) \geq J(C_{\text{ref}}^2)$.

Proof: Given C_{ref}^1 , suppose control $u^1(t)$ achieves the minimum cost $J(C_{\text{ref}}^1)$ and the corresponding states for the battery charge and capacity loss are $E_B^1(t)$ and $\Delta C^1(t)$. Since

$$E_B^1(t) + \Delta C^1(t) \leq C_{\text{ref}}^1 < C_{\text{ref}}^2,$$

$$\eta_B u^1(t) T_c + \Delta C^1(t) \leq C_{\text{ref}}^1 < C_{\text{ref}}^2 \text{ if } u^1(t) > 0,$$

$$-\frac{u^1(t)}{\eta_B} T_c + \Delta C^1(t) \leq C_{\text{ref}}^1 < C_{\text{ref}}^2 \text{ if } u^1(t) < 0,$$

$$P_{\text{load}}(t) - \eta_{\text{pv}}P_{\text{pv}}(t) + u^1(t) \leq D,$$

$u^1(t)$ is also a feasible control for problem (8) with C_{ref}^2 , and results in the cost $J(C_{\text{ref}}^1)$. Since $J(C_{\text{ref}}^2)$ is the minimum cost over the set of all feasible controls which include $u^1(t)$, we must have $J(C_{\text{ref}}^1) \geq J(C_{\text{ref}}^2)$. ■

In other words, J is non-increasing with respect to the parameter C_{ref} , i.e., J is monotonically decreasing (though may not be strictly monotonically decreasing). If $C_{\text{ref}} = 0$, then $0 \leq \Delta C(t) \leq C_{\text{ref}} = 0$, which implies that $u(t) = 0$. In this case, J has the largest value

$$J_{\text{max}} = J(0) = \int_{t_0}^{t_0+T} C_g(\tau)(P_{\text{load}}(\tau) - \eta_{\text{pv}}P_{\text{pv}}(\tau))d\tau. \quad (12)$$

Proposition 2 also justifies the storage size determination problem. Note that the critical value C_{ref}^c (as defined in Problem 1) is unique as shown below.

Proposition 3 Given the optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c , and D , C_{ref}^c is unique.

Proof: We prove it via contradiction. Suppose C_{ref}^c is not unique. In other words, there are two different critical values C_{ref}^{c1} and C_{ref}^{c2} . Without loss of generality, suppose $C_{\text{ref}}^{c1} < C_{\text{ref}}^{c2}$. By definition, $J(C_{\text{ref}}^{c1}) > J(C_{\text{ref}}^{c2})$ because C_{ref}^{c2} is a critical value, while $J(C_{\text{ref}}^{c1}) = J(C_{\text{ref}}^{c2})$ because C_{ref}^{c1} is a critical value. A contradiction. Therefore, we must have $C_{\text{ref}}^{c1} = C_{\text{ref}}^{c2}$. ■

Intuitively, if the unit cost for the battery capacity loss K is higher (compared with purchasing electricity from the grid), then it might be preferable that the battery is not used at all, which results in $C_{\text{ref}}^c = 0$, as shown below.

Proposition 4 Consider the optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c , and D under Assumption 1.

i) If

$$K \geq \frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z},$$

then $J(C_{\text{ref}}) = J_{\text{max}}$, which implies that $C_{\text{ref}}^c = 0$;

ii) if

$$K < \frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z},$$

then

$$J(C_{\text{ref}}) \geq J_{\text{max}} - (\max_t C_g(t) - \min_t C_g(t) - \frac{KZ}{\eta_B}) \times T \times (D + \max_t(\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))),$$

where \max_t and \min_t are calculated for $t \in [t_0, t_0 + T]$.

Proof: The cost function can be rewritten as

$$\begin{aligned} J(C_{\text{ref}}) &= J_{\text{max}} + \int_{t_0}^{t_0+T} C_g(\tau)u(\tau)d\tau + K\Delta C(t_0 + T) \\ &= J_{\text{max}} + \int_{t_0}^{t_0+T} C_g(\tau)u(\tau)d\tau + \\ &\quad K \int_{t_0}^{t_0+T} -Z \frac{u(\tau)}{\eta_B} |_{u(\tau) < 0} d\tau \\ &= J_{\text{max}} + J_+ + J_-, \end{aligned}$$

where

$$J_+ = \int_{t_0}^{t_0+T} C_g(\tau)u(\tau)|_{u(\tau) > 0} d\tau,$$

and

$$J_- = \int_{t_0}^{t_0+T} (C_g(\tau) - \frac{KZ}{\eta_B})u(\tau)|_{u(\tau) < 0} d\tau.$$

Note that $J_+ \geq 0$ because the integrand $C_g(\tau)u(\tau)|_{u(\tau) > 0}$ is always nonnegative.

i) We first consider $K \geq \frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z}$. There are two possibilities:

- $K \geq \frac{(\max_t C_g(t))\eta_B}{Z}$, or equivalently, $\max_t C_g(t) \leq \frac{KZ}{\eta_B}$. Thus, for any $t \in [t_0, t_0 + T]$, $C_g(t) - \frac{KZ}{\eta_B} \leq 0$, which implies that $J_- \geq 0$. Therefore, $J(C_{\text{ref}}) \geq J_{\text{max}}$.
- $\frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z} \leq K < \frac{(\max_t C_g(t))\eta_B}{Z}$. Let $A_1 = \max_t C_g(t) - \frac{KZ}{\eta_B}$, then $A_1 > 0$. Let $A_2 = \min_t C_g(t)$, then $A_2 \geq 0$. Since $\frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z} \leq K$, $A_2 \geq A_1$. Now we have $J_+ \geq A_2 \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau$, and $J_- \geq A_1 \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau$. Therefore,

$$\begin{aligned} J(C_{\text{ref}}) &\geq J_{\text{max}} + A_2 \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau + \\ &\quad A_1 \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau \\ &= J_{\text{max}} + (A_2 - A_1) \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau + \\ &\quad A_1 \int_{t_0}^{t_0+T} u(\tau)d\tau \\ &\geq J_{\text{max}} + A_1 \int_{t_0}^{t_0+T} u(\tau)d\tau. \end{aligned}$$

Since $E_B(t_0 + T) = E_B(t_0) + \int_{t_0}^{t_0+T} P_B(\tau)d\tau \geq 0$ and $E_B(t_0) = 0$, we have

$$\begin{aligned} 0 &\leq \int_{t_0}^{t_0+T} P_B(\tau)d\tau \\ &= \int_{t_0}^{t_0+T} P_B(\tau)|_{P_B(\tau) > 0} d\tau + \int_{t_0}^{t_0+T} P_B(\tau)|_{P_B(\tau) < 0} d\tau \\ &= \int_{t_0}^{t_0+T} u(\tau)\eta_B|_{u(\tau) > 0} d\tau + \int_{t_0}^{t_0+T} \frac{u(\tau)}{\eta_B}|_{u(\tau) < 0} d\tau \\ &\leq \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau + \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau \\ &= \int_{t_0}^{t_0+T} u(\tau)d\tau. \end{aligned} \quad (13)$$

Therefore, we have $J(C_{\text{ref}}) \geq J_{\text{max}}$.

In summary, if $K \geq \frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z}$, we have $J(C_{\text{ref}}) \geq J_{\text{max}}$, which implies that $J(C_{\text{ref}}) \geq J(0)$. Since $J(C_{\text{ref}})$ is a non-increasing function of C_{ref} , we also have $J(C_{\text{ref}}) \leq J(0)$. Thus, we must have $J(C_{\text{ref}}) = J(0) = J_{\text{max}}$ for $C_{\text{ref}} \geq 0$, and $C_{\text{ref}}^c = 0$ by definition.

ii) We now consider $K < \frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z}$, which implies that $A_1 > A_2 \geq 0$. Then

$$\begin{aligned} J(C_{\text{ref}}) &\geq J_{\text{max}} + A_2 \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau + \\ &\quad A_1 \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau \\ &= J_{\text{max}} + A_2 \int_{t_0}^{t_0+T} u(\tau)d\tau + \\ &\quad (A_1 - A_2) \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau. \end{aligned}$$

Since $\int_{t_0}^{t_0+T} u(\tau)d\tau \geq 0$ as argued in the proof to i), $J(C_{\text{ref}}) \geq J_{\text{max}} + (A_1 - A_2) \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau$. Now we try to lower bound $\int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau$. Note that $\int_{t_0}^{t_0+T} u(\tau)d\tau \geq 0$ (as shown in Eq. (13)) implies that

$$\int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau \geq - \int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau.$$

Given Assumption 1, S_3 is nonempty, which implies that $D + \max_t(\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)) > 0$. Since $u(t) \leq D + \eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t) \leq D + \max_t(\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))$, $\int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) > 0} d\tau \leq T \times (D + \max_t(\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)))$, which implies that

$$\int_{t_0}^{t_0+T} u(\tau)|_{u(\tau) < 0} d\tau \geq -T \times (D + \max_t(\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))). \quad (14)$$

In summary, $J(C_{\text{ref}}) \geq J_{\text{max}} - (A_1 - A_2) \times T \times (D + \max_t(\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)))$, which proves the result. ■

Given the result in Proposition 4, we impose the following additional assumption on the unit cost of the battery capacity loss to guarantee that C_{ref}^c is positive.

Assumption 2 In the optimization problem in Eq. (8),

$$K < \frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z}.$$

In other words, the cost of the battery capacity loss during operations is less than the potential gain expressed as the margin between peak and off-peak prices modified by the conversion efficiency of the battery converters and the battery aging coefficient.

Remark 2 There are two implications of Assumption 2:

- If the pricing signal is given, then the condition in Assumption 2 provides a criterion for evaluating the economic value of batteries compared to purchasing electricity from the grid. In other words, only if the unit cost of the battery capacity loss satisfies Assumption 2, it is desirable to use battery storages. For example, if the price is constant, then the condition reduces to $K < 0$, which cannot be satisfied by any battery. In other words, the use of batteries cannot reduce the total cost.
- If the battery and its converter are chosen, i.e., K, Z, η_B are all fixed, then the condition in Assumption 2 imposes a constraint on the pricing signal so that the battery can be used to lower the total cost. ■

Since $J(C_{\text{ref}})$ is a non-increasing function of C_{ref} and lower bounded by a finite value given Assumptions 1 and 2, the storage size determination problem is well defined. Now we show lower and upper bounds on the critical battery capacity in the following proposition.

Proposition 5 Consider the optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c , and D under Assumptions 1 and 2. Then $C_{\text{ref}}^{\text{lb}} \leq C_{\text{ref}}^c \leq C_{\text{ref}}^{\text{ub}}$, where

$$C_{\text{ref}}^{\text{lb}} = \max\left(\frac{T_c}{\eta_B} \times (\max_t (P_{\text{load}}(t) - \eta_{\text{pv}} P_{\text{pv}}(t)) - D), 0\right),$$

and

$$C_{\text{ref}}^{\text{ub}} = \max(\eta_B T_c + \frac{ZT}{\eta_B}, \eta_B T) \times (D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))).$$

Proof: We first show the lower bound via contradiction. Without loss of generality, we assume that $\frac{T_c}{\eta_B} \times (\max_t (P_{\text{load}}(t) - \eta_{\text{pv}} P_{\text{pv}}(t)) - D) > 0$ (because we require $C_{\text{ref}}^c \geq 0$). Suppose

$$C_{\text{ref}}^c < C_{\text{ref}}^{\text{lb}} = \frac{T_c}{\eta_B} \times (\max_t (P_{\text{load}}(t) - \eta_{\text{pv}} P_{\text{pv}}(t)) - D),$$

or equivalently,

$$D < -\frac{C_{\text{ref}}^c \eta_B}{T_c} + \max_t (P_{\text{load}}(t) - \eta_{\text{pv}} P_{\text{pv}}(t)).$$

Therefore, there exists $t_1 \in [t_0, t_0 + T]$ such that $D < -\frac{C_{\text{ref}}^c \eta_B}{T_c} + P_{\text{load}}(t_1) - \eta_{\text{pv}} P_{\text{pv}}(t_1)$. Since $u(t_1) \leq D - P_{\text{load}}(t_1) + \eta_{\text{pv}} P_{\text{pv}}(t_1) < -\frac{C_{\text{ref}}^c \eta_B}{T_c} \leq 0$, we have

$$\begin{aligned} P_B(t_1) &= \frac{u(t_1)}{\eta_B} \leq \frac{D + \eta_{\text{pv}} P_{\text{pv}}(t_1) - P_{\text{load}}(t_1)}{\eta_B} \\ &< -\frac{C_{\text{ref}}^c}{T_c} \leq -\frac{C_{\text{ref}}^c - \Delta C(t_1)}{T_c} = P_{\text{Bmin}}. \end{aligned}$$

The implication is that the control does not satisfy the discharging constraint at t_1 . Therefore, $C_{\text{ref}}^{\text{lb}} \leq C_{\text{ref}}^c$.

To show $C_{\text{ref}}^c \leq C_{\text{ref}}^{\text{ub}}$, it is sufficient to show that if $C_{\text{ref}} \geq C_{\text{ref}}^{\text{ub}}$, the electricity that can be stored never exceeds $C_{\text{ref}}^{\text{ub}}$.

Note that the battery charging is limited by P_{Bmax} , i.e., $P_B(t) = \eta_B u(t) \leq P_{\text{Bmax}}$. Now we try to lower bound P_{Bmax} .

$$P_{\text{Bmax}} = \frac{C_{\text{ref}} - \Delta C(t)}{T_c} \geq \frac{C_{\text{ref}} - \Delta C(t_0 + T)}{T_c},$$

in which the second inequality holds because $\Delta C(t)$ is a non-decreasing function of t . By applying the aging model in Eq. (3), we have

$$P_{\text{Bmax}} \geq \frac{C_{\text{ref}} - \Delta C(t_0 + T)}{T_c} = \frac{C_{\text{ref}} + \int_{t_0}^{t_0 + T} Z \frac{u(\tau)}{\eta_B} |_{u(\tau) < 0} d\tau}{T_c}.$$

Using Eq. (14), we have

$$P_{\text{Bmax}} \geq \frac{C_{\text{ref}} - \frac{ZT}{\eta_B} \times (D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)))}{T_c}.$$

Since $C_{\text{ref}} \geq C_{\text{ref}}^{\text{ub}}$, we have

$$\begin{aligned} P_{\text{Bmax}} &\geq \frac{(\max(\eta_B T_c + \frac{ZT}{\eta_B}, \eta_B T) - \frac{ZT}{\eta_B}) \times (D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)))}{T_c} \\ &\geq \eta_B (D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))). \end{aligned}$$

Therefore, $u(t) \leq D + \eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t) \leq D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)) \leq \frac{P_{\text{Bmax}}}{\eta_B}$, which implies that $P_B(t) = \eta_B u(t) \leq P_{\text{Bmax}}$. Thus, the only constraint on u related to the battery charging is $u(t) \leq D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))$. In turn, during the time interval $[t_0, t_0 + T]$, the maximum amount of electricity that can be charged is

$$\begin{aligned} \int_{t_0}^{t_0 + T} P_B(\tau) |_{P_B(\tau) > 0} d\tau &= \int_{t_0}^{t_0 + T} \eta_B u(\tau) |_{u(\tau) > 0} d\tau \\ &\leq \eta_B T \times (D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t))), \end{aligned}$$

which is less than or equal to $C_{\text{ref}}^{\text{ub}}$. In summary, if $C_{\text{ref}} \geq C_{\text{ref}}^{\text{ub}}$, the amount of electricity that can be stored never exceeds $C_{\text{ref}}^{\text{ub}}$, and therefore, $C_{\text{ref}}^c \leq C_{\text{ref}}^{\text{ub}}$. ■

Remark 3 As discussed in Proposition 1, if S_3 is empty, or equivalently, $(D + \max_t (\eta_{\text{pv}} P_{\text{pv}}(t) - P_{\text{load}}(t)) \leq 0$, then $C_{\text{ref}}^{\text{ub}} \leq 0$, which implies that $C_{\text{ref}}^c = 0$; this is consistent with the result in Proposition 1 when S_1 is empty. ■

IV. ALGORITHMS FOR CALCULATING C_{ref}^c

In this section, we study algorithms for calculating the critical battery capacity C_{ref}^c .

Given the storage size determination problem, one approach to calculate the critical battery capacity is by calculating the function $J(C_{\text{ref}})$ and then choosing the C_{ref} such that the conditions in Problem 1 are satisfied. Though C_{ref} is a continuous variable, we can only pick a finite number of C_{ref} 's and then approximate the function $J(C_{\text{ref}})$. One way to pick these values is that we choose C_{ref} from $C_{\text{ref}}^{\text{ub}}$ to $C_{\text{ref}}^{\text{lb}}$ with the step size⁵ $-\tau_{\text{cap}} < 0$. In other words,

$$C_{\text{ref}}^i = C_{\text{ref}}^{\text{ub}} - i \times \tau_{\text{cap}},$$

⁵Note that one way to implement the critical battery capacity in practice is to connect multiple identical batteries of fixed capacity C_{fixed} in parallel. In this case, τ_{cap} can be chosen to be C_{fixed} .

where⁶ $i = 0, 1, \dots, L$ and $L = \lceil \frac{C_{\text{ref}}^{\text{ub}} - C_{\text{ref}}^{\text{lb}}}{\tau_{\text{cap}}} \rceil$. Suppose the picked value is C_{ref}^i , and then we solve the optimization problem in Eq. (8) with C_{ref}^i . Since the battery dynamics and aging model are nonlinear functions of $u(t)$, we introduce a binary indicator variable $I_{u(t)}$, in which $I_{u(t)} = 0$ if $u(t) \geq 0$ and $I_{u(t)} = 1$ if $u(t) < 0$. In addition, we use δt as the sampling interval, and discretize Eqs. (2) and (3) as

$$E_B(k+1) = E_B(k) + P_B(k)\delta t,$$

$$\Delta C(k+1) = \begin{cases} \Delta C(k) - ZP_B(k)\delta t & \text{if } P_B(k) < 0 \\ \Delta C(k) & \text{otherwise.} \end{cases}$$

With the indicator variable $I_{u(t)}$ and the discretization of continuous dynamics, the optimization problem in Eq. (8) can be converted to a mixed integer programming problem with indicator constraints (such constraints are introduced in CPLEX [22]), and can be solved using the CPLEX solver [22] to obtain $J(C_{\text{ref}}^i)$. In the storage size determination problem, we need to check if $J(C_{\text{ref}}^i) = J(C_{\text{ref}}^c)$, or equivalently, $J(C_{\text{ref}}^i) = J(C_{\text{ref}}^{\text{ub}})$; this is because $J(C_{\text{ref}}^c) = J(C_{\text{ref}}^{\text{ub}})$ due to $C_{\text{ref}}^c \leq C_{\text{ref}}^{\text{ub}}$ and the definition of C_{ref}^c . Due to numerical issues in checking the equality, we introduce a small constant $\tau_{\text{cost}} > 0$ so that we treat $J(C_{\text{ref}}^i)$ the same as $J(C_{\text{ref}}^{\text{ub}})$ if $J(C_{\text{ref}}^i) - J(C_{\text{ref}}^{\text{ub}}) < \tau_{\text{cost}}$. Similarly, we treat $J(C_{\text{ref}}^i) > J(C_{\text{ref}}^{\text{ub}})$ if $J(C_{\text{ref}}^i) - J(C_{\text{ref}}^{\text{ub}}) \geq \tau_{\text{cost}}$. The detailed algorithm is given in Algorithm 1. At Step 4, if $J(C_{\text{ref}}^i) - J(C_{\text{ref}}^0) \geq \tau_{\text{cost}}$, or equivalently, $J(C_{\text{ref}}^i) - J(C_{\text{ref}}^{\text{ub}}) \geq \tau_{\text{cost}}$, we have $J(C_{\text{ref}}^i) > J(C_{\text{ref}}^{\text{ub}})$. Because of the monotonicity property in Proposition 2, we know $J(C_{\text{ref}}^j) \geq J(C_{\text{ref}}^i) > J(C_{\text{ref}}^{\text{ub}}) = J(C_{\text{ref}}^c)$ for any $j = i + 1, \dots, L$. Therefore, the **for** loop can be terminated, and the approximated critical battery capacity is C_{ref}^{i-1} .

It can be verified that Algorithm 1 stops after at most $L + 1$ steps, or equivalently, after solving at most

$$\lceil \frac{C_{\text{ref}}^{\text{ub}} - C_{\text{ref}}^{\text{lb}}}{\tau_{\text{cap}}} \rceil + 1$$

optimization problems in Eq. (8). The accuracy of the critical battery capacity is controlled by the parameters $\delta t, \tau_{\text{cap}}, \tau_{\text{cost}}$. Fixing $\delta t, \tau_{\text{cost}}$, the output is within

$$[C_{\text{ref}}^c - \tau_{\text{cap}}, C_{\text{ref}}^c + \tau_{\text{cap}}].$$

Therefore, by decreasing τ_{cap} , the critical battery capacity can be approximated with an arbitrarily prescribed precision.

Since the function $J(C_{\text{ref}})$ is a non-increasing function of C_{ref} , we propose Algorithm 2 based on the idea of bisection algorithms. More specifically, we maintain three variables

$$C_{\text{ref}}^1 < C_{\text{ref}}^3 < C_{\text{ref}}^2,$$

in which C_{ref}^1 (or C_{ref}^2) is initialized as $C_{\text{ref}}^{\text{lb}}$ (or $C_{\text{ref}}^{\text{ub}}$). We set C_{ref}^3 to be $\frac{C_{\text{ref}}^1 + C_{\text{ref}}^2}{2}$. Due to Proposition 2, we have

$$J(C_{\text{ref}}^1) \geq J(C_{\text{ref}}^3) \geq J(C_{\text{ref}}^2).$$

If $J(C_{\text{ref}}^3) = J(C_{\text{ref}}^2)$ (or equivalently, $J(C_{\text{ref}}^3) = J(C_{\text{ref}}^{\text{ub}})$); this is examined in Step 7), then we know that $C_{\text{ref}}^1 \leq C_{\text{ref}}^c \leq C_{\text{ref}}^3$;

⁶The ceiling function $\lceil x \rceil$ is the smallest integer which is larger than or equal to x .

Algorithm 1 Simple Algorithm for Calculating C_{ref}^c

Input: The optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c, D under Assumptions 1 and 2, the calculated bounds $C_{\text{ref}}^{\text{lb}}, C_{\text{ref}}^{\text{ub}}$, and parameters $\delta t, \tau_{\text{cap}}, \tau_{\text{cost}}$

Output: An approximation of C_{ref}^c

- 1: Initialize $C_{\text{ref}}^i = C_{\text{ref}}^{\text{ub}} - i \times \tau_{\text{cap}}$, where $i = 0, 1, \dots, L$ and $L = \lceil \frac{C_{\text{ref}}^{\text{ub}} - C_{\text{ref}}^{\text{lb}}}{\tau_{\text{cap}}} \rceil$;
 - 2: **for** $i = 0, 1, \dots, L$ **do**
 - 3: Solve the optimization problem in Eq. (8) with C_{ref}^i , and obtain $J(C_{\text{ref}}^i)$;
 - 4: **if** $i \geq 1$ and $J(C_{\text{ref}}^i) - J(C_{\text{ref}}^0) \geq \tau_{\text{cost}}$ **then**
 - 5: Set $C_{\text{ref}}^c = C_{\text{ref}}^{i-1}$, and exit the **for** loop;
 - 6: **end if**
 - 7: **end for**
 - 8: Output C_{ref}^c .
-

Algorithm 2 Efficient Algorithm for Calculating C_{ref}^c

Input: The optimization problem in Eq. (8) with fixed t_0, T, K, Z, T_c, D under Assumptions 1 and 2, the calculated bounds $C_{\text{ref}}^{\text{lb}}, C_{\text{ref}}^{\text{ub}}$, and parameters $\delta t, \tau_{\text{cap}}, \tau_{\text{cost}}$

Output: An approximation of C_{ref}^c

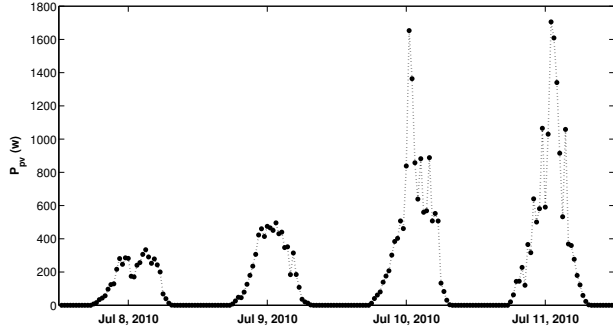
- 1: Let $C_{\text{ref}}^1 = C_{\text{ref}}^{\text{lb}}$ and $C_{\text{ref}}^2 = C_{\text{ref}}^{\text{ub}}$.
 - 2: Solve the optimization problem in Eq. (8) with C_{ref}^2 , and obtain $J(C_{\text{ref}}^2)$;
 - 3: Let sign = 1;
 - 4: **while** sign = 1 **do**
 - 5: Let $C_{\text{ref}}^3 = \frac{C_{\text{ref}}^1 + C_{\text{ref}}^2}{2}$;
 - 6: Solve the optimization problem in Eq. (8) with C_{ref}^3 , and obtain $J(C_{\text{ref}}^3)$;
 - 7: **if** $J(C_{\text{ref}}^3) - J(C_{\text{ref}}^2) < \tau_{\text{cost}}$ **then**
 - 8: Set $C_{\text{ref}}^2 = C_{\text{ref}}^3$, and $J(C_{\text{ref}}^2) = J(C_{\text{ref}}^3)$;
 - 9: **else**
 - 10: Set $C_{\text{ref}}^1 = C_{\text{ref}}^3$ and set $J(C_{\text{ref}}^1)$ with $J(C_{\text{ref}}^3)$;
 - 11: **if** $C_{\text{ref}}^2 - C_{\text{ref}}^3 < \tau_{\text{cap}}$ **then**
 - 12: Set sign = 0;
 - 13: **end if**
 - 14: **end if**
 - 15: **end while**
 - 16: Output $C_{\text{ref}}^c = C_{\text{ref}}^2$.
-

therefore, we update C_{ref}^2 with C_{ref}^3 but do not update⁷ the value $J(C_{\text{ref}}^2)$. On the other hand, if $J(C_{\text{ref}}^3) > J(C_{\text{ref}}^2)$, then we know that $C_{\text{ref}}^2 \geq C_{\text{ref}}^c \geq C_{\text{ref}}^3$; therefore, we update C_{ref}^1 with C_{ref}^3 and set $J(C_{\text{ref}}^1)$ with $J(C_{\text{ref}}^3)$. In this case, we also check if $C_{\text{ref}}^2 - C_{\text{ref}}^3 < \tau_{\text{cap}}$: if it is, then output C_{ref}^2 since we know the critical battery capacity is between C_{ref}^3 and C_{ref}^2 ; otherwise, the **while** loop is repeated. Since every execution of the **while** loop halves the interval $[C_{\text{ref}}^1, C_{\text{ref}}^2]$ starting from $[C_{\text{ref}}^{\text{lb}}, C_{\text{ref}}^{\text{ub}}]$, the maximum number of executions of the **while** loop is $\lceil \log_2 \frac{C_{\text{ref}}^{\text{ub}} - C_{\text{ref}}^{\text{lb}}}{\tau_{\text{cap}}} \rceil$, and the algorithm requires solving at most

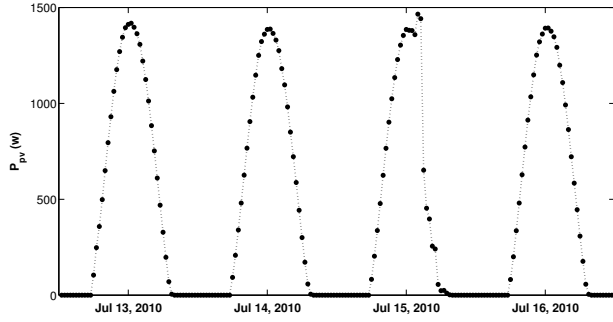
$$\lceil \log_2 \frac{C_{\text{ref}}^{\text{ub}} - C_{\text{ref}}^{\text{lb}}}{\tau_{\text{cap}}} \rceil + 1$$

optimization problems in Eq. (8). This is in contrast to solving $\lceil \frac{C_{\text{ref}}^{\text{ub}} - C_{\text{ref}}^{\text{lb}}}{\tau_{\text{cap}}} \rceil + 1$ optimization problems in Eq. (8) using Algorithm 1.

⁷Note that if we update $J(C_{\text{ref}}^2)$ with $J(C_{\text{ref}}^3)$, then the difference between $J(C_{\text{ref}}^2)$ and $J(C_{\text{ref}}^{\text{ub}})$ can be amplified when C_{ref}^2 is updated again later on.



(a) Starting from July 8, 2010.



(b) Starting from July 13, 2010.

Fig. 2. PV output based on GHI measurement at La Jolla, California, where tick marks indicate noon local standard time for each day.

Remark 4 Note that the critical battery capacity can be implemented by connecting batteries with fixed capacity in parallel because we only assume that the minimum battery charging time is fixed. ■

V. SIMULATIONS

In this section, we calculate the critical battery capacity using Algorithm 2, and verify the results in Section III via simulations. The parameters used in Section II are chosen based on typical residential home settings and commercial buildings.

A. Setting

The GHI data is the measured GHI in July 2010 at La Jolla, California. In our simulations, we use $\eta = 0.15$, and $S = 10m^2$. Thus $P_{pv}(t) = 1.5 \times GHI(t)(W)$. We have two choices for t_0 :

- t_0 is 0000 h local standard time (LST) on Jul 8, 2010, and the PV output is given in Fig. 2(a) for the following four days starting from t_0 (the interval is 30 minutes; this corresponds to the scenario in which there are relatively large variations in the PV output);
- t_0 is 0000 h LST on Jul 13, 2010, and the PV output is given in Fig. 2(b) for the following four days starting from t_0 (this corresponds to the scenario in which there are relatively small variations in the PV output).

The time-of-use electricity purchase rate $C_g(t)$ is

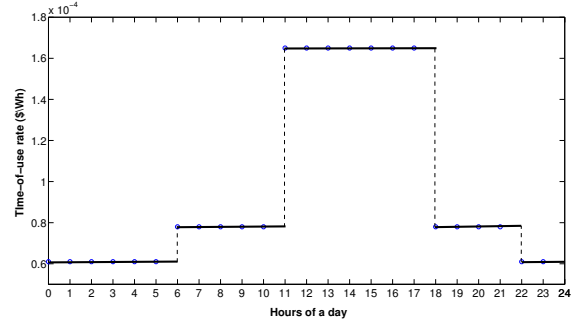


Fig. 3. Time-of-use pricing for the summer season at San Diego, CA [18].

- $16.5\text{¢}/kWh$ from 11AM to 6PM (on-peak);
- $7.8\text{¢}/kWh$ from 6AM to 11AM and 6PM to 10PM (semi-peak);
- $6.1\text{¢}/kWh$ for all other hours (off-peak).

This rate is for the summer season proposed by SDG&E [18], and is plotted in Fig. 3.

Since the battery dynamics and aging are characterized by continuous ordinary differential equations, we use $\delta t = 0.5h$ as the sampling interval, and discretize Eqs. (2) and (3) as

$$E_B(k+1) = E_B(k) + P_B(k)\delta t,$$

$$\Delta C(k+1) = \begin{cases} \Delta C(k) - ZP_B(k)\delta t & \text{if } P_B(k) < 0 \\ \Delta C(k) & \text{otherwise.} \end{cases}$$

In simulations, we assume that lead-acid batteries are used; therefore, the aging coefficient is $Z = 3 \times 10^{-4}$ [5], the unit cost for capacity loss is $K = 0.15\$/Wh$ based on the cost of $150\$/kWh$ [23], and the minimum charging time is $T_c = 12h$ [24].

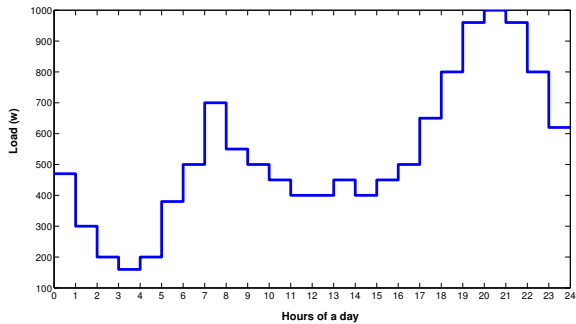
For the PV DC-to-AC converter and the battery DC-to-AC/AC-to-DC converters, we use $\eta_{pv} = \eta_B = 0.9$. It can be verified that Assumption 2 holds because the threshold value for K is⁸

$$\frac{(\max_t C_g(t) - \min_t C_g(t))\eta_B}{Z} = \frac{(16.5 - 6.1) \times 10^{-5} \times 0.9}{3 \times 10^{-4}} \approx 0.3120.$$

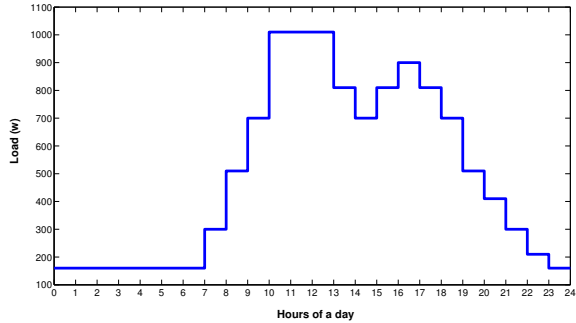
For the load, we consider two typical load profiles: the residential load profile as given in Fig. 4(a), and the commercial load profile as given in Fig. 4(b). Both profiles resemble the corresponding load profiles in Fig. 8 of [17].⁹ Note that in the residential load profile, one load peak appears in the early morning, and the other in the late evening; in contrast, in the commercial load profile, the two load peaks appear during the daytime and occur close to each other. For multiple day simulations, the load is periodic based on the load profiles in Fig. 4.

⁸Suppose the battery is a Li-ion battery with the same aging coefficient as a lead-acid battery. Since the unit cost $K = 1.333\$/Wh$ based on the cost of $1333\$/kWh$ [23] (and $K = 0.78\$/Wh$ based on the 10-year projected cost of $780\$/kWh$ [23]), the use of such a battery is not as competitive as directly purchasing electricity from the grid.

⁹However, simulations in [17] start at 7AM so Fig. 4 is a shifted version of the load profile in Fig. 8 of [17].



(a) Residential load averaged at 536.8(W).



(b) Commercial load averaged at 485.6(W).

Fig. 4. Typical residential and commercial load profiles.

For the parameter D , we use $D = 800(W)$. Since the maximum of the loads in Fig. 4 is around $1000(W)$, we will illustrate the peak shaving capability of battery storage. It can be verified that Assumption 1 holds.

B. Results

We first examine the storage size determination problem using Algorithm 2 in the following setting (called the basic setting):

- the cost minimization duration is $T = 24(h)$,
- t_0 is 0000 h LST on Jul 13, 2010, and
- the load is the residential load as shown in Fig. 4(a).

The lower and upper bounds in Proposition 5 are calculated as $2667(Wh)$ and $39269(Wh)$. When applying Algorithm 2, we choose $\tau_{\text{cap}} = 10(Wh)$ and $\tau_{\text{cost}} = 10^{-4}$. When running the algorithm, 13 optimization problems in Eq. (8) have been solved. The maximum cost J_{max} is -0.1681 while the minimum cost is -0.3022 , which is larger than the lower bound -2.5242 as calculated based on Proposition 4. The critical battery capacity is calculated to be $16714(Wh)$.

Now we examine the solution to the optimization problem in Eq. (8) in the basic setting with three different battery capacities $C_{\text{ref}}^c = 16714(Wh)$, $C_{\text{ref}}^L = 20714(Wh)$ (namely, a battery capacity larger than the critical capacity), and $C_{\text{ref}}^S = 12714(Wh)$ (namely, a battery capacity smaller than the critical capacity).

For the value C_{ref}^c , we solve the mixed integer programming problem using the CPLEX solver [22], and the objective function is $J(16714) = -0.3022$. $P_B(t)$, $E_B(t)$ and $C(t)$ are plotted

in Fig. 5(a). The plot of $C(t)$ is consistent with the fact that there is capacity loss (i.e., the battery ages) only when the battery is discharged. The capacity loss is around $2.24(Wh)$, and

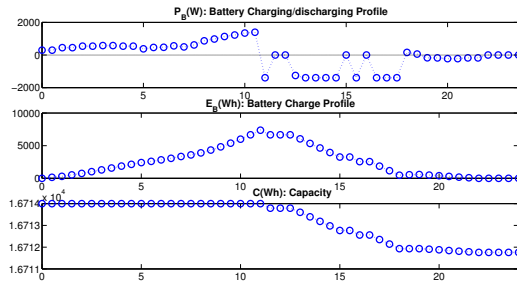
$$\frac{\Delta C(t_0 + T)}{C_{\text{ref}}} = \frac{2.24}{16714} = 1.34 \times 10^{-4} \approx 0,$$

which justifies the assumption we make when linearizing the nonlinear battery aging model in the Appendix. The dynamic pricing signal $C_g(t)$, $P_B(t)$, $P_{\text{load}}(t)$ and $P_g(t)$ are plotted in Fig. 5(b). Now we examine the effects of power arbitrage. From 8AM to 11AM, there is surplus PV generation as shown in Fig. 5(c). However, the surplus generation is not sold back to the grid (as $P_g(t) > 0$ during this period from the third plot of Fig. 5(b)) because currently the electricity price is not high enough. Instead it is stored in the battery (which can be observed from the second plot of Fig. 5(b)), and is then sold when the price is high after 11AM. From the third plot in Fig. 5(b), it can be verified that, to minimize the cost, electricity is purchased from the grid when the time-of-use pricing is low, and is sold back to the grid when the time-of-use pricing is high; in addition, the peak demand in the late evening (that exceeds $D = 800(W)$) is shaved via battery discharging, as shown in detail in Fig. 5(d).

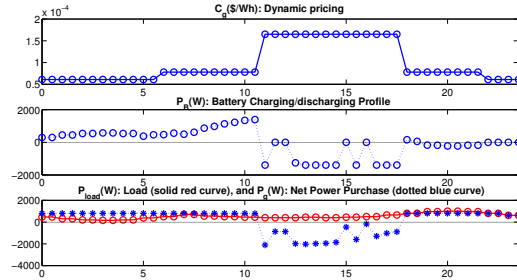
For the larger battery capacity $C_{\text{ref}}^L = 20714(Wh)$, we solve the mixed integer programming problem, and the objective function is $J(20714) = -0.3022$, which is the same as the cost with C_{ref}^c as expected. $P_B(t)$, $E_B(t)$ and $C(t)$ are plotted in Fig. 6(a). The dynamic pricing signal $C_g(t)$, $P_B(t)$, $P_{\text{load}}(t)$ and $P_g(t)$ are plotted in Fig. 6(b). It can be observed that in this case $P_B(t)$ and $P_g(t)$ are different from the ones with the battery capacity C_{ref}^c even though the costs are the same. The implication is that optimal control to the problem in Eq. (8) is not necessarily unique. Similarly, for the smaller battery capacity $C_{\text{ref}}^S = 12714(Wh)$, we solve the mixed integer programming problem, and the objective function is $J(12714) = -0.2950$, which is larger than the cost with C_{ref}^c as expected. $P_B(t)$, $E_B(t)$ and $C(t)$ are plotted in Fig. 7(a). The dynamic pricing signal $C_g(t)$, $P_B(t)$, $P_{\text{load}}(t)$ and $P_g(t)$ are plotted in Fig. 7(b). Based on the plots of $P_g(t)$ in Fig. 5(b) and Fig. 7(b), it can be observed that in this case, the amount of electricity sold back to the grid is smaller than the case with C_{ref}^c , which results in a larger cost.

Those observations of the case with the battery capacity C_{ref}^c for $T = 24(h)$ also hold for $T = 48(h)$. In this case, we change T to be $48(h)$ in the basic setting, and solve the battery sizing problem. The critical battery capacity is calculated to be $C_{\text{ref}}^c = 16800(Wh)$. Now we examine the solution to the optimization problem in Eq. (8) with the critical battery capacity $16800(Wh)$, and obtain $J(16800) = -0.5657$. $P_B(t)$, $E_B(t)$, $C(t)$ are plotted in Fig. 8(a), and the dynamic pricing signal $C_g(t)$, $P_B(t)$, $P_{\text{load}}(t)$ and $P_g(t)$ are plotted in Fig. 8(b). Note that the battery is gradually charged in the first half of each day, and then gradually discharged in the second half to be empty at the end of each day, as shown in the second plot of Fig. 8(a).

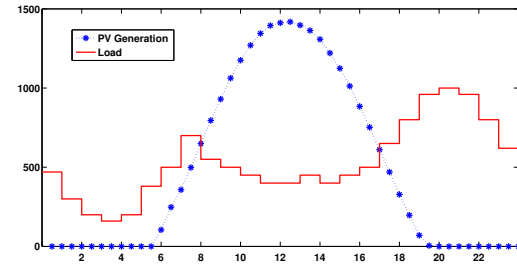
To illustrate the peak shaving capability of the dynamic pricing signal as discussed in Remark 1, we change the load



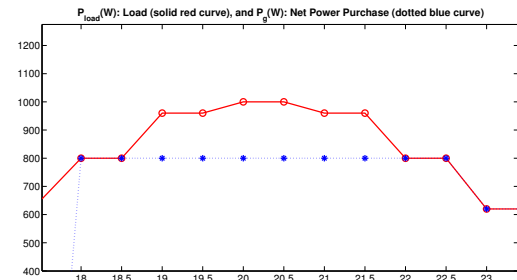
(a)



(b)



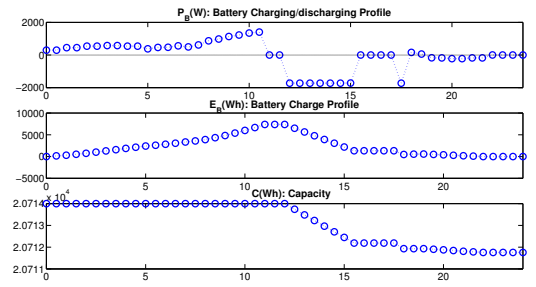
(c)



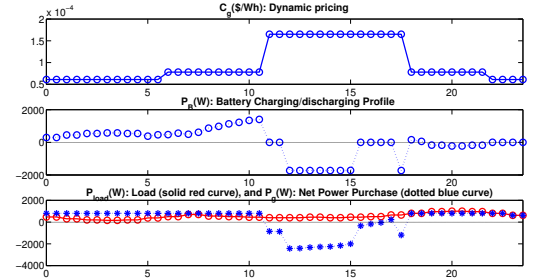
(d)

Fig. 5. Solution to a typical setting in which t_0 is on Jul 13, 2010, $T = 24(h)$, the load is shown in Fig. 4(a), and $C_{\text{ref}} = 16714(Wh)$.

to the commercial load as shown in Fig. 4(b) in the basic setting, and solve the battery sizing problem. The critical battery capacity is calculated to be $C_{\text{ref}}^c = 13816(Wh)$, and $J(13816) = -0.1596$. The dynamic pricing signal $C_g(t)$, $P_B(t)$, $P_{\text{load}}(t)$ and $P_g(t)$ are plotted in Fig. 9. For the commercial load, the duration of the peak loads coincides with that of the high price. To minimize the total cost, during peak times the battery is discharged, and the surplus electricity from PV after supplying the peak loads is sold back to the grid resulting in a negative net power purchase from the grid,

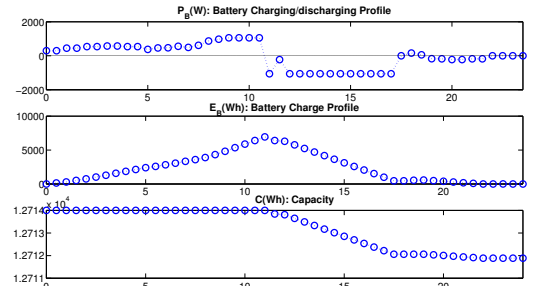


(a)

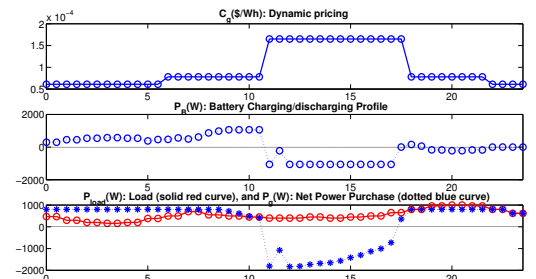


(b)

Fig. 6. Solution to a typical setting in which t_0 is on Jul 13, 2010, $T = 24(h)$, the load is shown in Fig. 4(a), and $C_{\text{ref}} = 20714(Wh)$.



(a)



(b)

Fig. 7. Solution to a typical setting in which t_0 is on Jul 13, 2010, $T = 24(h)$, the load is shown in Fig. 4(a), and $C_{\text{ref}} = 12714(Wh)$.

as shown in the third plot in Fig. 9. Therefore, unlike the residential case, the high price indirectly forces the shaving of the peak loads.

Now we consider settings in which the load could be either residential loads or commercial loads, the starting time could be on either Jul 8 or Jul 13, 2010, and the cost optimization duration can be $24(h)$, $48(h)$, $96(h)$. The results are shown in

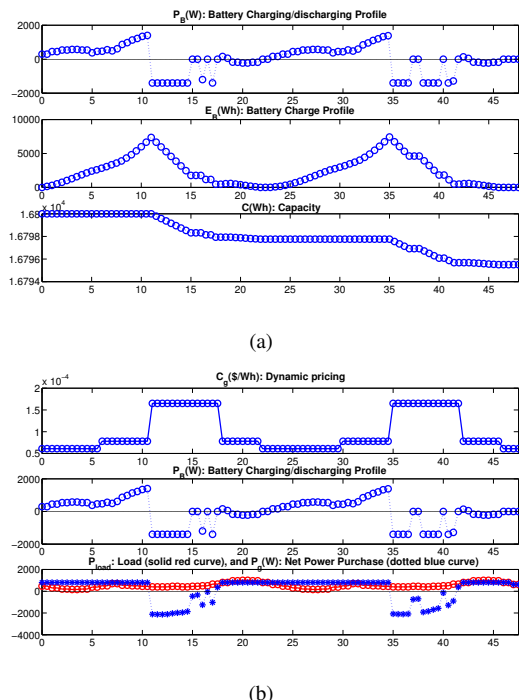


Fig. 8. Solution to a typical setting in which t_0 is on Jul 13, 2010, $T = 48(h)$, the load is shown in Fig. 4(a), and $C_{\text{ref}} = 16800(Wh)$.

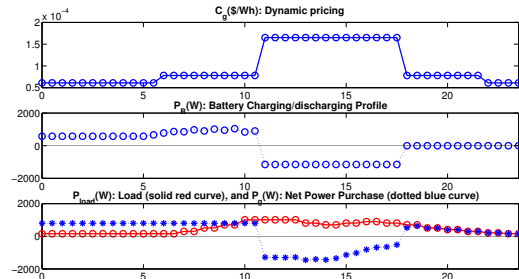


Fig. 9. Solution to a typical setting in which t_0 is on Jul 13, 2010, $T = 24(h)$, the load is shown in Fig. 4(b), and $C_{\text{ref}} = 13816(Wh)$.

Tables I and II. In Table I, t_0 is on Jul 8, 2010, while in Table II, t_0 is on Jul 13, 2010. In the pair $(24, R)$, 24 refers to the cost optimization duration, and R stands for residential loads; in the pair $(24, C)$, C stands for commercial loads.

We first focus on the effects of load types. From Tables I and II, the commercial load tends to result in a higher cost (even though the average of the commercial load is smaller than that of the residential load) because the peaks of the commercial load coincide with the high price. Commercial loads tend to result in larger optimum battery capacity C_{ref}^c as shown in Tables I and II, presumably because the peak load occurs during the peak pricing period and reductions in surplus PV production have to be balanced by additional battery capacity. If t_0 is on Jul 8, 2010, the PV generation is relatively lower than the scenario in which t_0 is on Jul 13, 2010, and as a result, the battery capacity is smaller and the cost is higher. This is because it is more profitable to store PV generated electricity than grid purchased electricity. From Tables I and II, it can be observed that the cost optimization

duration has relatively larger impact on the battery capacity for commercial loads, and relatively less impact for residential loads.

In Tables I and II, the row J_{max} corresponds to the cost in the scenario without batteries, the row $J(C_{\text{ref}}^c)$ corresponds to the cost in the scenario with batteries of capacity C_{ref}^c , and the row *Savings*¹⁰ corresponds to $J_{\text{max}} - J(C_{\text{ref}}^c)$. For Table I, we can also calculate the relative percentage of savings using the formula $\frac{J_{\text{max}} - J(C_{\text{ref}}^c)}{J_{\text{max}}}$, and get the row *Percentage*. One observation is that the relative savings by using batteries increase as the cost optimization duration increases. For example, when $T = 96(h)$ and the load type is residential, 15.60% cost can be saved when a battery of capacity 9461(Wh) is used; when $T = 96(h)$ and the load type is commercial, 25.05% cost can be saved when a battery of capacity 14088(Wh) is used. This clearly shows the benefits of utilizing batteries in grid-connected PV systems. In Table II, only the absolute savings are shown since negative costs are involved.

VI. CONCLUSIONS

In this paper, we studied the problem of determining the size of battery storage for grid-connected PV systems. We proposed lower and upper bounds on the storage size, and introduced an efficient algorithm for calculating the storage size. Batteries are used for power arbitrage and peak shaving. Note that, when PV generation costs reach grid parity, abundant PV generation will cause demand peaks and high electricity price at night, and low prices will occur during the day. Under this scenario, batteries could also be used for energy shifting, i.e., saving energy when PV generation is larger than load during the day for a future time when load exceeds PV generation at night.

In our analysis, the conversion efficiency of the PV DC-to-AC converter and the battery DC-to-AC and AC-to-DC converters is assumed to be a constant. We acknowledge that this is not the case in the current practice, in which the efficiency of converters depends on the input power in a nonlinear fashion [5]. This will be part of our future work. Another implicit assumption we made is that there is no energy loss due to battery self-discharging. Current investigation on battery self-discharging (e.g., the work in [25], [26]) shows that the self-discharging rate is very small on the order of 0.2% per day. In addition, such work is usually based on electric circuit equivalent models and assumes that a battery is not used for a long time, both of which do not hold in our setting. We leave this issue as part of our future work. In addition, we would like to extend the results to distributed renewable energy storage systems, and generalize our setting by taking into account stochastic PV generation.

APPENDIX

Derivation of the Simplified Battery Aging Model

Eqs. (11) and (12) in [5] are used to model the battery capacity loss, and are combined and rewritten below using the notation in this work:

$$C(t+\delta t) - C(t) = -C_{\text{ref}} \times X \times \left(\frac{E_B(t)}{C(t)} - \frac{E_B(t+\delta t)}{C(t+\delta t)} \right). \quad (15)$$

¹⁰Note that in Table II, part of the costs are negative. Therefore, the word ‘‘Earnings’’ might be more appropriate than ‘‘Savings’’.

TABLE I
SIMULATION RESULTS FOR t_0 ON JUL 8, 2010

	(24, R)	(48, R)	(96, R)	(24, C)	(48, C)	(96, C)
C_{ref}^c	6885	8173	9461	8827	13217	14088
$J(C_{\text{ref}}^c)$	0.8480	1.5646	2.0234	0.9890	1.7930	2.4272
J_{max}	0.9308	1.7406	2.3975	1.1410	2.1610	3.2384
Savings	0.0828	0.1760	0.3741	0.1520	0.3680	0.8112
Percentage	8.90%	10.11%	15.60%	13.32%	17.03%	25.05%

TABLE II
SIMULATION RESULTS FOR t_0 ON JUL 13, 2010

	(24, R)	(48, R)	(96, R)	(24, C)	(48, C)	(96, C)
C_{ref}^c	16714	16800	16847	13816	17691	17618
$J(C_{\text{ref}}^c)$	-0.3022	-0.5657	-0.9096	-0.1596	-0.3357	-0.5045
J_{max}	-0.1681	-0.2938	-0.3644	0.0421	0.1267	0.4766
Savings	0.1341	0.2719	0.5452	0.2017	0.4624	0.9811

If δt is very small, then $C(t + \delta t) \approx C(t)$. Therefore, $\frac{E_B(t)}{C(t)} - \frac{E_B(t+\delta t)}{C(t+\delta t)} \approx \frac{-P_B(t)\delta t}{C(t)}$. If we plug in the approximation, divide δt on both sides of Eq. (15), and let δt goes to 0, then we have

$$\frac{dC(t)}{dt} = C_{\text{ref}} \times Z \times \frac{P_B(t)}{C(t)}. \quad (16)$$

This holds only if $P_B(t) < 0$ as in [5], i.e., there could be capacity loss only when discharging the battery.

Since Eq. (16) is a nonlinear equation, it is difficult to solve. Let $\Delta C(t) = C_{\text{ref}} - C(t)$, then Eq. (16) can be rewritten as

$$\frac{d\Delta C(t)}{dt} = -Z \times \frac{P_B(t)}{1 - \frac{\Delta C(t)}{C_{\text{ref}}}}. \quad (17)$$

If t is much shorter than the life time of the battery, then the percentage of the battery capacity loss $\frac{\Delta C(t)}{C_{\text{ref}}}$ is very close to 0. Therefore, Eq. (17) can be simplified to the following linear ODE

$$\frac{d\Delta C(t)}{dt} = -Z \times P_B(t),$$

when $P_B(t) < 0$. If $P_B(t) \geq 0$, there is no capacity loss, i.e.,

$$\frac{dC(t)}{dt} = 0.$$

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Sonia Martinez (M'01, SM'07) is an associate professor at the Mechanical and Aerospace Engineering department at UC San Diego. She received her Ph.D. degree in Engineering Mathematics from the Universidad Carlos III de Madrid, Spain, in May 2002. Following a year as a Visiting Assistant Professor of Applied Mathematics at the Technical University of Catalonia, Spain, she obtained a Postdoctoral Fulbright fellowship and held positions as a visiting researcher at UIUC and UCSB. Dr. Martinez' main research interests include nonlinear control theory, cooperative control and networked control systems. In particular, her work has focused on the modeling and control of robotic sensor networks, the development of distributed coordination algorithms for groups of autonomous vehicles, and the geometric control of mechanical systems. She is also interested in control applications in energy systems. For her work on the control of underactuated mechanical systems she received the Best Student Paper award at the 2002 IEEE Conference on Decision and Control. She was the recipient of a NSF CAREER Award in 2007. For the paper "Motion coordination with Distributed Information," coauthored with Francesco Bullo and Jorge Cortes, she received the 2008 Control Systems Magazine Outstanding Paper Award.



Yu Ru (S'06, M'10) is a postdoctoral researcher at the Mechanical and Aerospace Engineering department at UC San Diego. He received the B.E. degree in Industrial Automation in 2002, and the M.S. degree in Control Theory and Engineering in 2005, both from Zhejiang University, Hangzhou, China, and the Ph.D. degree in Electrical and Computer Engineering in 2010 from University of Illinois at Urbana-Champaign, Urbana. His research focuses on optimization of grid-connected renewable energy systems, coordination control of multi-agent systems, estimation, diagnosis, and control of discrete event systems, and Petri net theory. He was the recipients of the prestigious Robert T. Chien Memorial Award and Sundaram Seshu International Student Fellowship from ECE at University of Illinois at Urbana-Champaign.



Jan Kleissl is an assistant professor at the Dept. of Mechanical and Aerospace Engineering at the University of California, San Diego (UCSD) and Associate Director, UCSD Center for Energy Research. Kleissl received a Ph.D. in 2004 from Johns Hopkins University in Environmental Engineering and joined UC San Diego in 2006. Kleissl supervises 12 PhD students who work on solar power forecasting, solar resource model validation, and solar grid integration work funded by DOE, CPUC, NREL, and CEC. Kleissl teaches classes in Renewable Energy Meteorology, Fluid Mechanics, and Laboratory Techniques at UC San Diego. Kleissl received the 2009 NSF CAREER Award and 2008 Hellman Fellowship for tenure-track faculty of great promise, the 2008 UC San Diego Sustainability Award.