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Traffic Delay Reduction via Distributed Dynamic Lane Reversal and Rerouting

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***Abstract*— Traffic congestion is a major source of delays in modern road networks. Motivated by this, in this paper we propose two distributed algorithms to reduce delays: a dynamic lane reversal algorithm and a rerouting algorithm. When there is a large density of vehicles on one side of a road and a small density on the other, time can be saved by reallocating lanes from the less dense side to the more dense side, which motivates dynamic lane reversal. When a road has greater density than nearby roads, time can be saved by redirecting flow into the least congested roads, this motivates dynamic rerouting. Given a communication system between infrastructure and vehicles on the road, the local state of the network can be approximated and utilized by the algorithms to minimize travel time. Equilibrium conditions for the system are analyzed, convergence of the lane reversal algorithm to a critical point is proved, and overall performance is examined in simulation.**

I. INTRODUCTION

Motivation. Congestion is a major source of traffic delays in modern road networks, and the problem is growing. Significant imbalances of traffic density in a given road network can arise due to many events, such as when there is a large flow of vehicles towards an industrial center in the morning, a large event ends and there is a mass of flow out from large event, or there is an accident which creates heavy congestion on one side of a road, for example. Modern infrastructure endowed with new information technology requires no additional space or construction, and can substantially reduce overall traffic delays. Motivated by this, here we investigate the implementation and benefits of lane reversal and traffic rerouting distributed algorithms that can improve traffic flow.

In particular, recent advances in design, performance, and cost of autonomous vehicles (see [5]) has fueled a growing interest in Autonomous Intersection Management (AIM), an efficient policy for coordinating autonomous vehicles using an intersection manager (IM) to safely pass through an intersection [9]. With the help of the AIM policy and vehicle-to-infrastructure communications, an approximation of the state of traffic can be constructed. The IM can then implement more dynamic procedures to reverse one or more lanes or communicate a new route to some vehicles if traffic delays will be reduced. The future presence of autonomous vehicles is also important in implementing the actual lane reversal and vehicle rerouting, as physically moving a barrier to reverse a lane is a slow process that can take hours, [1], yet merely indicating a lane's direction or a new route for a vehicle is likely to cause driver confusion and increase risk of accident. With advances in vehicle autonomy, lane reversal and rerouting are less restricted by physical safety considerations and can be achieved through simple communication from the traffic signal to the vehicle.

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Literature review. Many recent papers have furthured Autonomous Intersection Management. Batch processing of reservations in AIM to enforce liveness is proposed in [3]. An auction-based scheme under AIM is analyzed in [6]. Local information is shared and utilized to minimize delay time under Greenshield's traffic model in [20]. Some effort has also recently been put towards solving vehicle routing problems in modern context. A provably safe distributed solution for coordinating vehicles outside an intersection is provided in [16]. Work has also been done analyzing traffic evolution over networks. Classical traffic models are examined in a network setting in [19]. Passivity is used to generalize the network flow control problem in [18]. A solution to the problem of assigning freight loads to available carriers given unbalanced network conditions is found in [2].

Much of the literature concerning lane reversal discusses evacuation procedures in order to respond effectively to natural disasters, [7], [17]. These papers propose the solution of lane reversal to accommodate emergency evacuation in a non-dynamic way. Some works discuss procedures and statistics for location-specific cases where lane reversal would be beneficial, [22], [21]. More recently, some have attempted to further improve results through dynamic lane reversal. The solution presented in [11] requires a centralized computer to find an allocation strategy, with a minimum timestep of one hour. In [13], the authors formulate a model and present a centralized solution which does not use network dynamics. In [10], dynamic lane reversal is implemented on a single road and tested in simulation.

Statement of contributions. In this paper we extend the cell transmission model to characterize the evolution of vehicle density in a road network and the effect of both lane reversal and rerouting on these dynamics. We establish objective functions with the goal of minimizing total vehicle time spent on the road, and propose two algorithms. Using lane reversal, we propose a distributed algorithm to efficiently calculate and implement an appropriate lane allocation and prove convergence of the algorithm to a more efficient solution. We analyze the long term behavior of the system of a road network with balanced lanes, and establish its convergence to an equilibrium under certain regularity conditions of its sources, sinks, and traffic on intersections. Then, we propose a distributed rerouting algorithm to more efficiently achieve this long-term equilibrium. We show through simulations performance gains using lane reversal and rerouting on various initial conditions.

Notation and Nomenclature. We let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_{\geq 0}$ denote the set of positive real numbers, and $\mathbb{Z}$ denote the set of integers. Similarly, $\mathbb{R}^n$ (resp. $\mathbb{R}^n_{\geq 0}, \mathbb{Z}^n$) denotes the product space of n copies of $\mathbb{R}$ (resp. $\mathbb{R}_{\geq 0}, \mathbb{Z}$). The vector of ones with length n is denoted by $\mathbf{1}_n$. A *directed graph* G consists of a set of *vertices* V and a set of *directed edges* E, $G = (V, E)$, such that $E \subset V \times V$. Vertex a is an *out-neighbor* of vertex b if $(b, a) \in E$. Similarly, a is an *in-neighbor* of b if $(a, b) \in E$. Vertex a is a neighbor of b if b is an out-neighbor or in-neighbor of a. The set of out-neighbors (resp. in-neighbors) of a is denoted $\mathcal{N}_a^{\text{out}}$ (resp. $\mathcal{N}_a^{\text{in}}$). We say that matrix $A = \{a_{ij}\}$ is $A \in \text{sparse}(G)$, for $G = (V, E)$, if $a_{ij} = 0$ when $(i, j) \notin E$. Given a vertex set V, V_r denotes the set of cells contained on road r.

II. Problem Statement

We consider traffic evolving over a road network. Each road consists of one or two *sides* for each direction of traffic flow and which have a given number of lanes. In addition, each side is divided into *cells* of length L, which are used to describe the evolution of traffic density, see Figure 1.

We define a directed graph $G_{\text{C}} = (C, E_{\text{C}})$ of cells $i \in C$, such that $(j, h) \in E_{\text{C}}$ if traffic can flow from cell j to cell h. A *road* is defined as the set of connected cells bounded on each side by a source, sink, or an intersection manager (IM). A source (resp. a sink) is a special cell in which traffic only flows out (resp. flows in,) while an IM is an intelligent traffic management system at an intersection of roads. We denote by R the set of all roads and $\mathcal{N}_r$ the set of neighbors of road r, where two roads are neighbors if they share an intersection. We denote S as the set of all road sides, $p, -p \in S$ are the two sides of a road, $n = |C|$, and $s = |S|$. The intersection graph $G_{\text{Z}} = (Z, E_{\text{Z}})$ consists of the vertex set Z containing all IMs and edges $(z_1, z_2) \in Z$ if there is exactly one road connecting intersections z_1 and $z_2 \in Z$. A cell which flows into a sink is denoted by $i \in \underline{B}$, and a cell which receives flow from a source is denoted by $\overline{B}$.

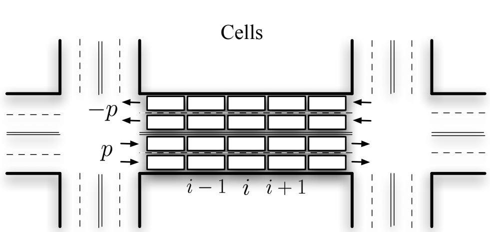


Fig. 1: Road divided into cells

A. Traffic Model

The following traffic model is based on the *Lighthill-Whitham-Richards Partial Differential Equation*, [12] and [15], to describe the evolution of traffic density on each road side, $\rho : \mathbb{R}_{\geq 0} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$,

$$\partial_t \rho + \partial_x Q(\rho) = 0. \tag{1}$$

This equation maintains conservation of mass, and the flow function $Q(\rho)$ is given by

$$Q(\rho) = \begin{cases} v_f \rho, & \rho \leq \rho_c, \\ \frac{v_f \rho_c}{\rho_{\text{jam}} - \rho_c}(\rho_{\text{jam}} - \rho), & \rho > \rho_c, \end{cases} \tag{2}$$

where v_f is the free flow speed of the vehicles, ρ_{jam} is the density at which a traffic jam occurs, and ρ_c is the critical density value where maximum flow occurs, see Figure 2. This model is based on experimental data and is commonly used to model traffic flow, particularly because it captures the wave behavior of traffic.

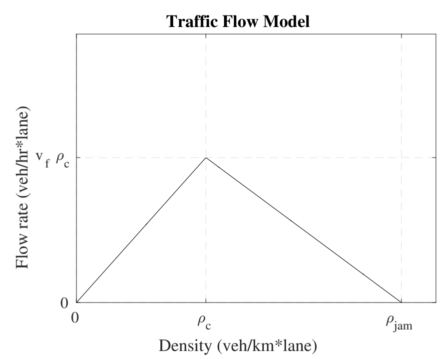


Fig. 2: Vehicle flow model

The *Cell Transmission Model* [8] is a discretization of (1) using time step Δt and spatial step Δx, where is assumed that all cells have length $L = \Delta x$. For a cell i with exactly one in-neighbor $i-1$ and one out-neighbor $i+1$, the density of the cell is updated according to

$$\rho_i(t+1) = \rho_i(t) + \frac{\Delta t}{L(\ell_p + u_p)}(q(\rho_{i-1}(t), \rho_i(t)) - q(\rho_i(t), \rho_{i+1}(t))),$$

where i is contained on side p of road r, ℓ_p is the number of default lanes of side p, $\rho_i(t)$ is the density (vehicles/lane-km) of vehicles on i at time t, and $u_p(t) \in \{1-\ell_p, \ldots, \ell_{-p}-1\}$ is the number of additional lanes on side p. The constraint $u_p + u_{-p} = \ell_p + \ell_{-p}$ must hold to keep the total number of lanes in a road constant, where $u_{-p} \in \{\ell_{-p}-1, \ldots, \ell_p - 1\}$ is the number of additional lanes on side $-p$. We have

$$q(\rho_{i-1}(t), \rho_i(t)) = q_{i-1,i}(t) = \min\{\mathbf{q}_{i-1}(t), \mathbf{q}_i(t)\}, \tag{3}$$

is the flow rate (vehicles/hr) from $i-1$ to i. The piecewise functions $\mathbf{q}_{i-1}(t)$ and $\mathbf{q}_i(t)$ are defined as

$$\mathbf{q}_{i-1}(t) = \begin{cases} v_f(\ell_p + u_p(t))\rho_{i-1}(t), & \rho_{i-1}(t) \leq \rho_c, \\ v_f(\ell_p + u_p(t))\rho_c, & \rho_{i-1}(t) > \rho_c, \end{cases}$$

$$\mathbf{q}_i(t) = \begin{cases} v_f(\ell_p + u_p(t))\rho_c, & \rho_i(t) \leq \rho_c, \\ \frac{v_f \rho_c}{\rho_{\text{jam}} - \rho_c}(\ell_p + u_p(t))(\rho_{\text{jam}} - \rho_i(t)), & \rho_i(t) > \rho_c. \end{cases}$$

Intuitively, the flow from $i-1$ to i is restricted when $\rho_{i-1}(t)$ is small or $\rho_i(t)$ is large [4].

Cells can also be connected to sources or sinks of various strengths, these make up the boundary to the system. In this paper, we define the flow from a source b to cell $i \in C$ on side $p \in S$ to be

$$q_{b,i}(t) = \begin{cases} \alpha_b(t) v_f(\ell_p + u_p(t))\rho_c, & \rho_i(t) \leq \rho_c, \\ \frac{\alpha_b(t) v_f \rho_c}{\rho_{\text{jam}} - \rho_c}(\ell_p + u_p(t))(\rho_{\text{jam}} - \rho_i(t)), & \rho_i(t) > \rho_c, \end{cases}$$

where $\alpha_b(t) \in [0,1]$ is the strength of source b at time t. In turn, the flow from i in $p \in S$ to sink c is described by:

$$q_{i,c}(t) = \begin{cases} \beta_c(t) v_f(\ell_p + u_p(t))\rho_i(t), & \rho_i(t) \leq \rho_c, \\ \beta_c(t) v_f(\ell_p + u_p(t))\rho_c, & \rho_i(t) > \rho_c, \end{cases}$$

where $\beta_c(t) \in [0,1]$ is the strength of sink c at time t.

To model a network of roads at intersections, the flow out of a cell must equal the sum of flows into other cells. We define a matrix $K = \{k_{ij}\} \in \mathbb{R}^{n \times n}$ where k_{ij} contains the fraction of vehicles which move from cell i to cell j. For now, we assume K is constant. If j is the only out-neighbor of i in G_{C} then $k_{ij} = 1$, but if j is one of multiple out-neighbors, then $k_{ij} < 1$. The flow out of any cell i, based on conservation of mass, is given by

$$q_i^{\text{out}}(t) = \sum_{j \in \mathcal{N}_i^{\text{out}}} k_{ij} q_{i,j}(t), \tag{4}$$

where $\rho \in \mathbb{R}^n$ is the vector of cell densities which are not sources or sinks and $\mathcal{N}_i^{\text{out}}$ is the set of out-neighbors of i in G_{C}. In this model, intersections are assumed to be small compared to the length of each cell, so the time spent in the intersection is negligible. The role of an efficient Autonomous Intersection Management policy is important to this assumption.

We similarly define

$$q_i^{\text{in}}(t) = \sum_{h \in \mathcal{N}_i^{\text{in}}} k_{hi} q_{h,i}(t), \tag{5}$$

where $\mathcal{N}_i^{\text{in}}$ is the set of in-neighbors of i in G_{C}.

The evolution of any cell in the network is given by

$$\rho_i(t+1) = \rho_i(t) + \frac{\Delta t}{L(\ell_p + u_p)}(q_i^{\text{in}}(t) - q_i^{\text{out}}(t)), \quad i \in C. \tag{6}$$

Ultimately, the system can be written as

$$\rho(t+1) = A_{\sigma(\rho(t))}\rho(t) + b_{\sigma(\rho(t))}, \tag{7}$$

where $\sigma(\rho(t)) \in \overline{\sigma}$ is the mode determined by the density of the state, $\overline{\sigma}$ is the set of modes, and $A_{\sigma(\rho(t))}, b_{\sigma(\rho(t))}$ are constructed using all equations from (3) to (6). In the literature, this is known as the *Switching Mode Model* (SMM) [14].

For lane reversal, the control input $u \in \mathbb{Z}^{|R|}$ controls the number of lanes per road which directly affects that road's density, see Figures 3 and 4.

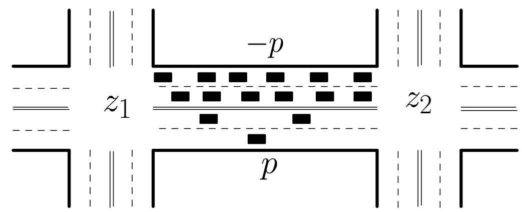


Fig. 3: Before lane reversal

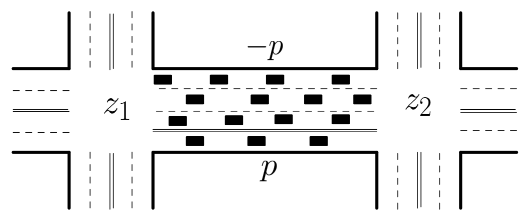


Fig. 4: After lane reversal

Based on conservation of mass, we update the density of each cell i on a road side p after lane reversal as follows:

$$\rho_i^+ = \rho_i \cdot \frac{\ell_p + u_p}{\ell_p + u_p^+},$$

where ρ_i is the density of i, u_p is the control before the discrete update to u_p^+, which results in an update ρ_i^+. In this paper we assume that the change in road density is instantaneous, based on an assumption that vehicles respond quickly to a lane opening or closing. The clearing time t_c, the time it takes for all vehicles to vacate a lane being reversed, is also assumed to be zero. Lane clearing can realistically be performed in 15 seconds or less under most traffic conditions in which a lane clearing occurs, so this assumption is reasonable, [10].

B. Problem Formulation

To characterize performance of the system, we define the objective function as the time spent of each vehicle in the system G_{C} summed over every vehicle, or

$$\overline{W}(u) = \sum_{w=1}^{N} \left(t_w^\ell - t_w^e\right),$$

where t_w^ℓ is the time in which vehicle w leaves G through a sink, t_w^e is the time in which vehicle w enters G through a source, and N is the total number of vehicles that spent time within the system. The total time spent is inversely proportional to the total flowrate, so total time can be approximated as

$$\begin{aligned}\overline{W}(u) &\approx \frac{N}{q_{\text{avg}} \sum_{p\in S} \ell_p}, \\ &\approx \sum_{t=0}^{t_f} \left(\frac{N}{\sum_{i\in C}(q_i^{\text{in}}(t) + q_i^{\text{out}}(t))/2} \right),\end{aligned}$$

where q_{avg} is the average flow rate in G.

We define our first control input as the directional lane allocation of each road $u \in \mathbb{Z}^{|R|}$, where $u_p = 1$ corresponds to reversing one lane from the default lanes in the direction of $-p$ to the direction of p in the road $r \in R$. The goal is to minimize $\overline{W}(u)$ while satisfying two physical constraints, one which maintains the total number of lanes of a roadway (the sum of lanes in both directions is constant), and the other which requires a positive integer number of lanes. This is stated as

Problem 1:

$$\begin{aligned}\underset{u\in\mathbb{R}^{|R|}}{\text{maximize}} \quad & W(u) = \sum_{t=0}^{t_f} \left(\sum_{i\in C} (q_i^{\text{in}}(t) + q_i^{\text{out}}(t)) \right) \\ \text{subject to} \quad & u_p \in \{-\ell_p + 1, \dots, \ell_{-p} - 1\}, \\ & u_{-p} \in \{-\ell_{-p} + 1, \dots, \ell_p - 1\}, \\ & u_p + u_{-p} = \ell_p + \ell_{-p}, \forall\, p, -p \in S.\end{aligned}$$

A point u^* is a critical point for Problem 1 if u^* satisfies the above constraints and if $W(u^*) \geq W(u)$ for all u s.t. $\forall\, p \in S, u$ satisfies the above constraints and $u^*_\zeta = u_\zeta$, $\forall\, \zeta \neq p$.

If vehicles can be redirected through intersections, then $K = \{k_{ij}\} \in \mathbb{R}^{n\times n}$ is our control variable, where k_{ij} is the proportion of vehicles flowing from cell i to cell j. Each non-zero value is lower bounded by a value $k_{\min}$ in order to maintain connectedness of the graph. Assuming a uniform critical density value in some network, we know maximum flow can be obtained when $\rho_i = \rho_c$, $\forall\, i \in C$, so to simplify we only look at the current time step and write this new problem as

Problem 2:

$$\begin{aligned}\underset{K\in\mathbb{R}^{n\times n}}{\text{minimize}} \quad & \tilde{W}(K) = \sum_{i\in C} |\rho_i(t+1) - \rho_c| \\ \text{subject to} \quad & (K\mathbf{1}_n)_i = 1, \forall\, i \notin \underline{B}, \\ & (K\mathbf{1}_n)_i = 0, \forall\, i \in \underline{B}, \\ & K \in \text{sparse}(G_C), \\ & k_{ij} \in [k_{\min}, 1], \forall\, (i,j) \in E.\end{aligned}$$

The best solution to Problem 2 will drive the state towards its highest long-term flow.

C. Approximation of the State by IMs

Here, we will use the assumptions employed in [10] for an IM to approximate the state of the traffic on the roads at the intersection. Vehicles have unique identifiers and transmit a message within $D \approx 300$ meters to the IM for a reservation request to cross intersections more efficiently. The IM at intersection $z \in Z$ maintains a counter variable $\overline{z}_p$ for road side p, adding one to $\overline{z}_p$ when it receives a notification message from a vehicle on road side p and subtracting one from $\overline{z}_p$ whenever a vehicle from road side p with a confirmed reservation is expected to leave the road and enter the intersection. The state of road side p at time t is calculated as

$$\rho_p(t) = \frac{\overline{z}_p(t)}{(\ell_p + u_p)\min\{L, D\}}. \tag{8}$$

If $L > D$ then assume that the state of the entire road is equal to the state in the local section. Note, with more sensing than just at intersections, the state of the roads can be more accurately approximated, so smaller cells can be used. This approximation is used in both algorithms to determine whether or not travel efficiency can be improved.

III. Lane Reversal Policy

In this section we provide a distributed Lane Reversal Algorithm together with its stability properties. The performance of the algorithm is also analyzed in Section V.

A. Lane Reversal Algorithm

Problem 1 is a non-convex, non-smooth optimization problem with integer constraints. To simplify it, we ignore the time horizon and optimize at each time step. We assume that there is an IM z_1 and z_2 at both ends of each road, and that z_1 is assigned its control. This IM requires estimates of the road states from its neighboring IMs and from the neighbors of z_2 to construct the complete local state. These estimates are calculated by counting vehicles in and out of each road as explained in Section II-C and in Equation (8).

We define $C_p = \{a \in C \mid a \text{ is a cell of } p\}$ for $p \in S$, and similarly $C_r = C_p \cup C_{-p}$, where $p, -p$ are the road sides of $r \in R$. Suppose that $T(t') \in \{1, \dots, \overline{T}\}$ represents a clock ticking from 1 to $\overline{T}$ at each IM synchronously, where $\overline{T}$ is the maximum number of ticks for which the algorithm will run and $\Delta t' < \Delta t$ is a smaller discrete time step. Each IM updates its assigned roads on specific ticks, which are given by a schedule $\Lambda(r) \in \{1, \dots, \overline{T}\}$, computed during an initialization phase. As an example, in the road network in Figure 5 each road with the same number Λ can update simultaneously. By means of the flag function "`to_update`," computations are reduced to cases when changes in the neighboring conditions can lead to non-trivial updates. When the turn of an IM to update takes place (line 5), then, in order to find the best control policy for some road r with sides $p, -p \in S$ while keeping other roads fixed,

$W_r(u + \omega_r \Delta_r)$ is maximized over $\omega_r \in \Omega_r$ in the LANE REVERSAL ALGORITHM. Here, $W_r = \sum_{a \in C_{r \cup \mathcal{N}_r}} (q_a^{\text{in}} + q_a^{\text{out}})$, $\Omega_r = \{-\ell_p + 1, \ldots, \ell_{-p} - 1\}$, u contains the set of current controls, and $\Delta_r \in \mathbb{Z}^n$ has zeros everywhere except 1 for each component $i \in C_p$ and -1 for each component $j \in C_{-p}$. Since in real road networks most roads have 4 or less lanes, an exhaustive search is computationally inexpensive in this domain. If a trivial update takes place, then a new update for neighboring roads is not necessary. This is encoded by setting the `to_update` function equal to zero, otherwise this function is set equal to one. State estimates are updated and information on updated controls, states and the `to_update` function is communicated to neighbors. The algorithm runs until time t_f.

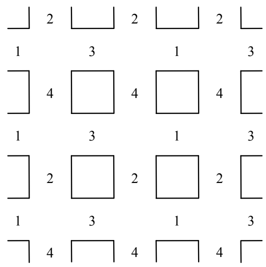


Fig. 5: Schedule of road network

Algorithm 1: Lane Reversal Algorithm of IM z

Initialize time $t' = 0$, schedule $\Lambda(r)$, $\forall\, r \in R$;
Initialize `to_update`$(r) = 1$, $\forall\, r \in R$;
for *all* $r \in R$ *with sides* $p, -p$ *controlled by* z *and* t' **do**
  Update $u_{p'}$, $u_{-p'}$, `to_update`(r'), and $\rho_{i'}$ if messages were received from neighbors;
  if *to_update* $= 1$ *and* $\Lambda(r) = T(t')$ **then**
    $\omega_r^* \leftarrow \text{argmin}_{\omega_r \in \Omega_r} W_r(u + \omega_r \Delta_r)$;
    $u_\nu^+ \leftarrow u_\nu$, $\forall\, \nu \in R \setminus \{p, -p\}$;
    $u_p^+ \leftarrow u_p + \omega_r^*$;
    $u_{-p}^+ \leftarrow u_p - \omega_r^*$;
    if $u_p^+ = u_p$ **then**
      `to_update`$(r) \leftarrow 0$;
    else
      `to_update`$(\lambda) \leftarrow 1$, $\forall\, \lambda \in \mathcal{N}_r$;
      `to_update`$(r) \leftarrow 0$;
    end
    Initiate lane swap, set $\rho_i^+ = \rho_i \cdot \frac{\ell_r + u_r}{\ell_r + u_r^+}$, $\forall\, i \in C_p$ and $\rho_j^+ = \rho_j \cdot \frac{\ell_r - u_r^+}{\ell_r - u_r}$, $\forall\, j \in C_{-p}$;
    Transmit u_p^+, u_{-p}^+, `to_update`$(\lambda) \forall\, \lambda \in \mathcal{N}_r$, $\rho_i^+ \forall\, i \in C_p$, and $\rho_i^+ \forall\, i \in C_{-p}$ values to neighbors of r;
  end
  $t' \leftarrow t' + \Delta t'$;
end

B. Stability Analysis of Lane Reversal

We first establish an upper bound on the objective function:

Lemma 3.1: Under the constraints given in Problem 1, the objective function satisfies

$$W(u) \leq 2 v_f \rho_c \ell n,$$

at each time step, assuming that $\ell_p = \ell$, $\forall\, p \in S$. This upper bound is achieved when roads are lane-balanced (for any path on the network, the number of lanes remains constant) and $\rho_i = \rho_c, \forall\, i \in C$. Intuitively, this is the state when flow through each lane is maximized over the whole network, and there is no congestion formed through lane merging. □

The proof for this lemma is omitted, as it is simply calculated by maximizing each cell's flow.

Lemma 3.2: The LANE REVERSAL ALGORITHM converges in finite time to a critical point u^* of Problem 1 under the listed constraints.

Proof: The update u^+ is implemented in the LANE REVERSAL ALGORITHM by evaluating $W_r(u)$ and choosing ω_r which maximizes this value. Note that the algorithm constrains u_p^+ s.t. $u_p^+ \in \{1 - \ell_p, \ldots, \ell_{-p} - 1\}$. Since a local maximizer of $W_r(u)$ also maximizes $W(u)$ and the algorithm maintains a schedule which is compatible with the separability of W (no two road neighbors update simultaneously), it is guaranteed that $W(u^+) \geq W(u)$. In this way, W is a monotonically non-decreasing function through the algorithm. Using a discrete-time Lyapunov stability argument with W, asymptotic convergence to a point u^* satisfying the constraints of Problem 1 for which W can not be improved by modifying u^* entry-wise can be guaranteed. Due to the finite discrete state space, convergence occurs in finite time. ■

IV. VEHICLE REROUTING POLICY

In this section, we provide a stability analysis for the road density evolving under the dynamics (3) to (6), under the assumption of balanced sources and sinks. This motivates the distributed rerouting algorithm, which is simulated in Section V. In this section, we assume that there is no lane reversal occuring, and that the number of lanes on each road are equal, so $\ell_p + u_p = \ell_{p'} + u_{p'} = \ell, \forall\, p, p' \in S$.

A. Stability Analysis of Weight-Balanced Traffic Networks

Because maximum flow is achieved when $\rho_i = \rho_c, \forall\, i \in C$, we are interested in finding conditions that guarantee that system achieves this state naturally. We find one such sufficient condition here.

We define $\alpha = [\alpha_1, \ldots, \alpha_n]^\top$, where $\alpha_i \in [0, 1]$ is the strength of the source connected to cell i ($\alpha_i = 0$ if no source is connected to cell i). Similarly, $\beta = [\beta_1, \ldots, \beta_n]^\top$ where $\beta_i \in [0, 1]$ is the strength of the sink connected to cell i ($\beta_i = 0$ if no sink is connected to cell i). We use two assumptions:

Assumption 4.1 (Critical Density Value): The critical density value is $\rho_c \in (0, \frac{\rho_{\text{jam}}}{2}]$.

Assumption 4.2 (Weight Balanced): The graph G_{C} is weight balanced with respect to the boundary conditions, or equivalently, $K^{\top}\mathbf{1}_n + \alpha = \mathbf{1}_n$ and $K\mathbf{1}_n + \beta = \mathbf{1}_n$.

Assumption 4.3 (Existence of Sources and Sinks): There exists a source and sink with nonzero strength somewhere in G_{C}.

Note that the first assumption may over-constrain the problem, depending on α and β and the specific example. However, under these conditions, the following result holds.

Theorem 4.4 (Stability to critical density values): Given Assumptions 4.1 on the value of ρ_c, 4.2 on the weight-balanced graph, 4.3 on the existence of sources and sinks, dynamics of Equations (3) through (6) and a connected graph G_{C}, any initial state with $\rho_i(0) \in [0, \rho_{\mathrm{jam}}], \forall\, i \in C$, converges practically to ρ_c. In other words, $\lim_{t\to\infty} \rho(t) \in [(\rho_c - \epsilon)\mathbf{1}_n, (\rho_c + \epsilon)\mathbf{1}_n]$, where $\epsilon \leq \frac{\Delta t v_f \rho_c}{L}$. □

Intuitively, this result holds due to the natural dynamics as well as the boundary conditions. The proof has been excluded due to space constraints.

We can make a similar Lyapunov argument proving that $\lim_{t\to\infty} \rho_i(t) \in [\rho_{\mathrm{src}}^{\min}, \rho_{\mathrm{snk}}^{\max}], \forall\, i \in C$, where $\rho_{\mathrm{src}}^{\min} = \rho_c \min_{i\in C}(\alpha_i + \sum_{j\in C} k_{ji})$ and $\rho_{\mathrm{snk}}^{\max} = \rho_{\mathrm{jam}} - \rho_c \min_{i\in C}(\beta_i + \sum_{j\in C} k_{ij})$. Bounding the flow rate in and out excludes any state from being at equilibrium outside these bounds, given the previous assumptions.

The previous analysis helps motivate the rerouting algorithm. Maintaining Assumption 4.2 is crucial for equilibrium behavior of the network, and is expected that, when boundary conditions oscillate about this condition, convergence to a close-to-equilibrium condition will occur. However, within these constraints we would like to hasten convergence to further reduce delays, the benefits of this are heightened under time-varying boundary conditions.

B. Rerouting Algorithm

In some situations, it is feasible to direct vehicles where to go, in order to maintain a balanced networks over time. One example is a factory setting where identical robots can have dynamically reassigned tasks. Freight-type of vehicles or autonomous cars in mobility-on-demand systems could also be influenced in real traffic based on AIM-vehicle communication mechanisms. Control over vehicle direction means that K can be altered under some constraints to improve the total flow over the time interval. Because maximum flow is achieved when $\rho_i = \rho_c, \forall\, i \in C$, flow through intersections can be redirected from more dense roads to less dense roads, keeping the system close to ρ_c. This problem is formulated in Problem 2.

A greedy approach suits this problem because it increases immediate flow through the intersection and speeds up the balancing of neighboring roads while maintaining the equilibrium of the natural dynamics. A potential strategy is to redirect flow from all in-neighbors of an intersection to the least dense out-neighbor. If the least dense out-neighbor is more dense than ρ_c, then redirect towards a sink if possible. However, this approach creates congestion when sinks are not ideal by attempting to direct flow out of the system but the sink restricting the flow out, so we require an alternative policy. Instead, each out-neighbor is sorted according to how much flow they will allow in, and they are paired with in-neighbors which provide the most flow, see REROUTING ALGORITHM.

Algorithm 2: Rerouting Algorithm

for *each intersection* $z \in Z$ *at each time* t **do**
    $D^+ \leftarrow$ vector of cells flowing into z, sorted by decreasing density;
    $D^- \leftarrow$ vector of cells receiving flow from z, sorted by increasing density;
    for $\gamma \in \{1, \ldots, |D^+|\}$ **do**
        $x \leftarrow D^+_\gamma$;
        $y \leftarrow D^-_\gamma$;
        for $\zeta \in \{1, \ldots, n\}$ **do**
            if $k_{x,\zeta} > 0$ *and* $\zeta \neq y$ **then**
                $k_{x,\zeta} \leftarrow k_{\min}$;
            end
            if $k_{\zeta,y} > 0$ *and* $\zeta \neq x$ **then**
                $k_{\zeta,y} \leftarrow k_{\min}$;
            end
        end
        $k_{x,y} \leftarrow 1 - (|D^+| - 1)k_{\min}$;
    end
end

This algorithm is decentralized, the only information required is from immediate neighbors of an IM. Intuitively, the algorithm is improving flow by directing flow from the most dense in-neighbors of the intersection to the least dense out-neighbors so that the flow from/to the most/least congested road is relatively unrestricted. This pushes both states more quickly towards ρ_c, and because they are the furthest away from ρ_c, this helps decrease $V(\rho(t))$ more rapidly. Under varying boundary conditions, this improvement is enhanced.

V. SIMULATION RESULTS

In simulation, both lane reversal and rerouting vehicles reduce overall traffic delay under imbalanced initial conditions as we discuss next. We use $\Delta t = 1$ second, $L = 500$ meters, $\ell = 4$ lanes per road, and $v_f = 60$ km/hr. Note, $\rho_{\mathrm{jam}} \approx 226 \frac{\text{vehicles}}{\text{km}\cdot\text{lane}}$ was calculated by assuming an average vehicle length of 4.11 meters and an average gap between stationary vehicles of 0.31 meters.

Lane Reversal Algorithm

Generally, lane reversal creates a significant short-term improvement of traffic flow. Lane reversal can hinder overall traffic flow when reversing a lane requires lane merging somewhere else in the system or when the road side with less lanes receives heavy traffic flow afterwards. This first issue can be addressed through coupling roads together so that their control variables are equal and no lane merging is required between them, in simulation we choose the more conservative control value and apply that to both roads. The second issue can be at least partially addressed through predicting traffic patterns using past data or communications with a larger portion of the road network, though we do not address this in this paper.

We simulated a two road network with an intersection between them, sources and sinks at the boundary, and an initial state which had heavy congestion randomly sampled from $[0, \rho_{\text{jam}}/2]$ on one side of both roads and light congestion randomly sampled from $\rho_{\text{jam}}/2, \rho_{\text{jam}}]$ on the other. The boundary conditions α and β on one end of the network were randomly sampled from $[0, 1]$ and $\alpha = 1$ and $\beta = 1$ on the other side, creating an imbalanced flow. U-turns are not allowed, and flow from any road to any neighbor is equally likely. The improvement in the objective function is shown in Table 6, we can see a large variance in the data but there is always significant improvement. One example of the benefit of lane reversal on the objective function is shown in Figure 7, the flowrate increases immediately after the lane reversal while maintaining the same equilibrium. Note, maintaining a constant number of lanes along all paths becomes impossible with a larger system, creating merges which negatively affect the overall equilibrium.

36.6	28.9	21.8	13.5	39.1
44.3	68.8	13.0	54.6	34.7

Fig. 6: Relative reversal improvement (%)

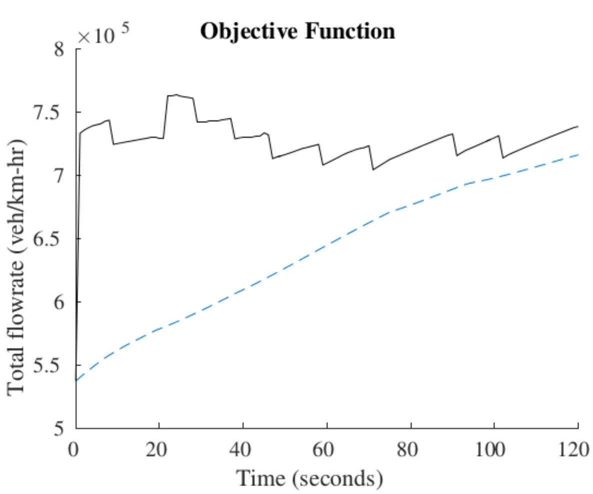
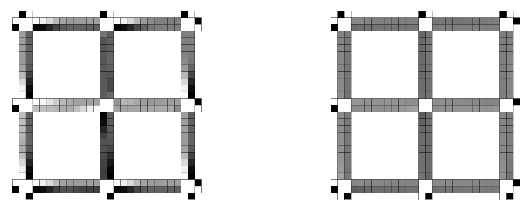


Fig. 7: Lane reversal on two roads

Rerouting Algorithm

The rerouting policy improves flow rate in both short term and long term while maintaining the original equilibrium, though in a less dramatic fashion than lane reversal. We have simulated a two block by two block road network with random initial densities under random constant boundary conditions with $k_{\text{min}} = 0.05$. We implemented the rerouting policy on the central intersection, see Figure 8, black represents very dense and white represents no vehicles. This shows an initially unbalanced state which converged to a more efficient state with help from the rerouting policy in the middle intersection.



$t = 0$ seconds $t = 20$ seconds

Fig. 8: Twelve Road Network

In Table 9, we can see relative improvements on the total flow rate given by this policy, ranging from negligible to moderate. We note that the objective function describes the performance of the each cell in the system, and in this network there are 240 cells, only 8 of which are connected to the intersection performing the policy, so moderate improvement is acceptable. Under specific initial conditions, the most improvement was seen at 36%, on average improvement is between 0 and 5%, see Figure 9. In each case, this algorithm improved overall flow rate.

1.3	0.1	5.7	0.1	1.3
3.3	0.0	3.4	4.6	3.9

Fig. 9: Relative rerouting improvement (%)

VI. Conclusion and Future Work

In conclusion, we extended the cell transmission model and established objective functions with the goal of minimizing total time spent on the road. We proposed a distributed algorithm to efficiently calculate and implement an appropriate lane direction reallocation. We also proposed a distributed algorithm to dynamically reroute vehicles to improve the long term behavior of the system. We proved convergence of the lane reversal algorithm to a critical point and bound the equilibria of the traffic rerouting algorithm under certain conditions. We showed through simulations performance gains using lane reversal on a network under particular conditions and using rerouting under different conditions.

There are many avenues for future work on this problem. One avenue is improving the traffic model and comparing its

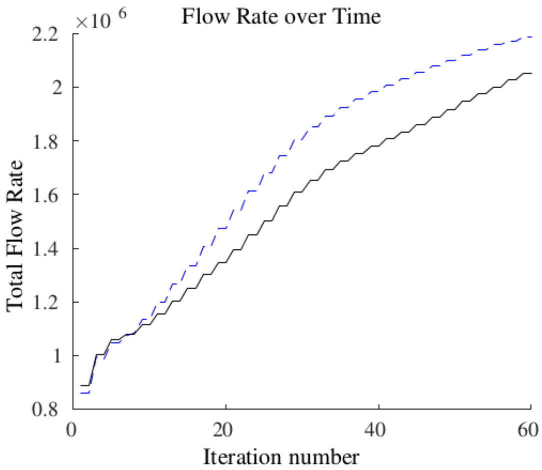


Fig. 10: Network Flow Rate

predictions with real traffic traffic data to ensure accuracy, in particular the vehicle time spent in an intersection is currently assumed constant. A microscopic model will better capture the dynamics of real vehicles on a road network, for example by implementing simulations which include spawning and tracking individual vehicles. Reinforcement learning can address some of the issues with using a greedy lane reversal algorithm to further reduce total time delays.

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