ON TRIGGERING CONTROL OF SINGLE-INPUT LINEAR SYSTEMS UNDER PULSE-WIDTH MODULATED DOS SIGNALS

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Abstract. This paper studies the stability of remotely controlled and observed single-input controllable linear systems under power-constrained Pulse-Width Modulated (PWM) Denial-of-Service (DoS) signals. The effect of a DoS jamming signal is to corrupt the communication channels, thus preventing the data to be received at its destination. In this work, we first assume that the DoS signal is *partially known*, i.e., a uniform lower-bound for the off time-intervals and the on-to-off transiting time-instants are known. We then propose a first type of resilient control and triggering strategies which are provably capable of beating partially known jamming signals. Building on this, we then present our joint control and identification algorithms, JAMCOID for Periodic Signals and JAMCOID, which are provably able to guarantee the system stability under *unknown* jamming signals. More precisely, JAMCOID for Periodic Signals algorithm is able to partly identify a periodic DoS signal with known uniform lower bound for the off time-intervals, whereas JAMCOID algorithm is capable of dealing with power-constrained, but otherwise unknown, DoS signals while ensuring stability. The practicality of the proposed techniques is evaluated on a simulation example under *partially known* and *unknown* jamming scenarios.

Key words. Linear systems; Nonlinear control; Triggering control; Denial-of-Service jamming signals; Cyber-physical systems

AMS subject classifications.

1. Introduction.

Cyber-physical systems comprise a wide range of systems that tightly integrate both computational and physical components. Thanks to new developments in sensing and communication technologies, these systems are being used in very diverse areas ranging from aerospace, to energy, to civil infrastructure facilities. While the benefits of cyber-physical systems are many, they also come at the price of several challenges. In particular, they are more broadly exposed to threats that can disrupt their normal operation. The latter has brought up and motivated renewed research on the topic of system resilience and security, see e.g. [8] and references therein. A specific type of threat arises from vulnerable communication links, which can be disrupted by means of viruses or external communication-signal jammers. Among these, *Denial-of-Service (DoS)* are reported to be the most common type of interference [7]. Motivated by their power-constrained nature, detection avoidance, and ease of implementation, DoS signals can further acquire *Pulse-Width Modulated (PWM)* signal pattern [20, 9]. In this work, we study how to adapt the control of a linear cyber-physical system to power-constrained PWM DoS jamming signals.

The secure operation of cyber-physical systems has been studied in different contexts. The papers [27, 25] characterize topological network conditions that allow a multi-agent system to detect other malicious agents injecting false data; while [3] studies how to maintain group connectivity despite the presence of malicious external jamming agents. On the other hand, the work [31] proposes a Receding Horizon control

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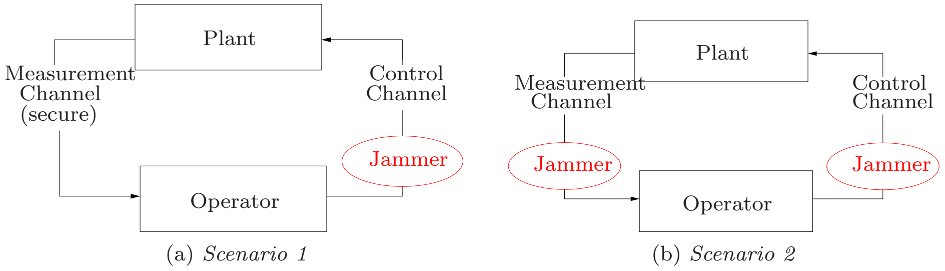
methodology to deal with a class of deceptive replay jammers, potentially introducing system delays in formation control missions. However, these previous works can only deal with simple dynamics for each agent (second-order integrators in [31]), and box type of state constraints at best. In [31] resilience comes at the expense of large receding horizons, which can be computationally expensive and difficult to implement.

Some representative studies in the context of Game Theory focus on malicious attacks on linear systems, leading to problem formulations that models the jammer and operator interactions as a dynamic zero-sum non-cooperative game. In this framework, one can single out [23], which consider power-constrained DoS jamming signals on discrete-time systems. The objective of this work is the characterization of equilibrium solutions for fixed-resource agents, which restricts the analytical results to one-dimensional control systems.

The problems of control and estimation over unreliable communication networks have received considerable attention over the last decade [18]. Topics of interest include quantization [6], delays [5], sampling [24], packet dropout [26], DoS jamming signals [1], and clock synchronization [16]. The DoS signals considered in [1] are modeled by means of a stochastic Bernoulli packet drop distribution. The goal is the minimization of a finite-horizon quadratic cost function subject to constraints. This work builds on previous research over lossy networks such as [26]. However, none of the aforementioned papers considers adaptation in the control law in order to exploit an energy limitation of the jamming signals. On estimation, the work [16] provides conditions under which synchronization of a affine-clock network subject to delays is possible. The method assumes information about the clock times is submitted in messages, and does not address how to estimate clocks while maintaining economic communications for an underlying system control. Finally, in the context of discrete-time linear systems, one can also distinguish [11] on deceptive jammers. Using sensor redundancy and compressed sensing techniques, the authors propose an encoding algorithm that can be resilient to this type of attacks. The algorithm does not account for possible communication interruptions as those imposed by DoS signals.

Motivated by the emerging use of economic communications in modern control systems, we address the problem of maintaining system stability in the context of triggering control [28, 22, 30]. In other words, we aim to build triggering control actions which rely on limited communications and/or measurements and which are then more robust with respect to a class of DoS PWM jamming signals. In this regard, the works [29, 17] present sufficient conditions on the maximum number of successive data dropouts that guarantee that a distributed system employing an event-triggering algorithm maintains stability. However, communications are not adapted to deal with any type of DoS signal. Finally, the paper [19] considers a resilience problem formulated in the triggering framework. This latter, deals with an alternative type of deceptive signals, which tamper with the control commands. Resilience is based on the switching between a safe and faulty modes to maintain normal system operation at all times. In this setting, the detection of the malfunction above a threshold is always possible, and then the attack has a limited effect on the system performance.

We consider three problem scenarios of increasing difficulty with respect to the assumed knowledge on the DoS signal. First, we consider a partially known PWM DoS jamming signal where the on-to-off time jamming instants are known as well as a guaranteed off period. In this setting, we present control triggering strategies that can be tuned arbitrarily to deal with any jammer of this type. Building upon



(a) *Scenario 1* (b) *Scenario 2*

Fig. 2.1: *Problem Architecture. Two scenarios have been considered: in Scenario 1 measurements are secure or indirectly available to the operator, and in Scenario 2 both the measurement and control channels can be compromised by a DoS signal.*

these results, we consider a second setting, where the jamming signal is assumed to be (non-necessarily malicious) periodic but of unknown period. To address this case, we introduce the JAMCOID for Periodic Signals algorithm that exploits periodicity to both synchronize while sporadically sample the jamming signal, and stabilize the system. Finally, in the third problem scenario we consider an unknown, but power-constrained, PWM DoS jamming signal. For this case, we propose the JAMCOID algorithm, which bestows a joint control and identification strategies at the expense of a higher number of communications. In these three problems, we prove that our proposed strategies ensure the system's asymptotic stability. In contrast with our earlier work [13], the contributions of this research study may be itemized as follows: (i) proposing a resilient parameter-dependent control and triggering strategies capable of dealing with the partially known jamming scenario, (ii) proposing the JAMCOID for Periodic Signals and JAMCOID algorithms to address the unknown jamming scenario, (iii) simulations on the functionality of both aforementioned contributions. A preliminary version of this work focusing on known jammers and systems of low dimension has appeared in [12]. The other preliminary version, entailing MIMO systems has appeared in [14], where the detailed proofs are omitted.

The rest of the paper is organized as follows. Section 2 includes the problem formulation and notations. Section 3 provides some preliminaries, along with our resilient control and strategy consistent with the jamming signal. In Section 4, we analyze and prove the stability of the system equipped with these resilient control and triggering strategies. In what follows in Section 5, we describe the jammer control and identification algorithms, JAMCOID for Periodic Signals and JAMCOID, and analyze their asymptotic behavior to prove that they guarantee the system stability. In Section 6, we illustrate the functionality of our theoretical results under known and unknown jamming scenarios. At last, in Section 7, we summarize results and state future work objectives.

2. Problem Formulation. We consider a remote operator-plant setup, where the operator uses control and measurement channels to send and receive data from an unstable plant. The wireless communication channels can be subject to jamming as depicted in Figure 2.1. We assume that the plant has no specific intelligence and is only able to update the control based on the data it receives. We also assume that the operator knows the plant dynamics and is able to compute and send the control and obtain its state measurements at particular times.

More precisely, we have:

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{2.1a}$$

$$u(t) = Kx(t_k)\,, \quad \forall t \in [t_k, t_{k+1}[\,, \tag{2.1b}$$

where $x \in \mathbb{R}^d$ is the state vector, $u \in \mathbb{R}$ is the input, A, B and K are matrices of proper dimensions, and $\{t_k\}_{k\in\mathbb{N}}$ is a certain triggering time sequence. Here, (i) the system (2.1a) is open-loop unstable, and (ii) the pair (A, B) is controllable.

We consider a type of *power-constrained* jamming signal or jammer, blocking the communication channels as follows, see Figure 2.2:

$$u_{\text{jmd}}(t) = \begin{cases} 0, & T^{n-1} \le t \le T^{n-1} + T^{n-1}_{\text{off}}, \\ 1, & T^{n-1} + T^{n-1}_{\text{off}} < t < T^{n}, \end{cases} \tag{2.2}$$

where the sequences of real numbers, $\{T^n\}_{n\in\mathbb{Z}}$, $\{T^n_{\text{off}}\}_{n\in\mathbb{Z}}$, satisfy $T^n < T^{n+1}$, $T^n_{\text{off}} \in \mathbb{R}_{>0}$, and $T^{n-1}_{\text{off}} < T^n - T^{n-1}$, for $n \in \mathbb{Z}$. Using these parameters, the intervals $[T^n, T^n + T^n_{\text{off}}]$ determine when the signal is off and communication is possible. We further denote by $\{T^n_{\text{on}}\}_{n\in\mathbb{Z}}$, with $T^n_{\text{on}} \in \mathbb{R}$, and $]T^n_{\text{on}}, T^{n+1}[$ the time interval where the jammer is active, thus no data can be transmitted. It holds that $T^n_{\text{off}} + T^n_{\text{on}} = T^n - T^{n-1}$, $\forall n$. The parameters T^n and T^n_{off} need not be time-invariant which recalls Pulse-Width Modulated (PWM) signals. Finally, $T^{\text{cr}}_{\text{off}}$ is a uniform lower-bound for T^n_{off}, i.e., $0 < T^{\text{cr}}_{\text{off}} \le T^n_{\text{off}}$, $\forall n$, while $T^{\text{cr,n}}_{\text{on}} \triangleq T^n - T^{n-1} - T^{\text{cr}}_{\text{off}}$. In addition, we assume $T^{\text{cr}}_{\text{off}} < \infty$ and $\{T^{\text{cr,n}}_{\text{on}}\} < \infty$, $\forall n \in \mathbb{N}$, these assumptions further justify the power-constrained nature of the DoS signal (2.2) because $\frac{T^n_{\text{on}}}{T^n_{\text{off}}} \le \frac{\{T^{\text{cr,n}}_{\text{on}}\}}{T^{\text{cr}}_{\text{off}}} < \infty$ holds. The last notation, for the case of $T^n = nT$, implies $T^{\text{cr,n}}_{\text{on}} = T - T^{\text{cr}}_{\text{off}}$, so we use $T^{\text{cr}}_{\text{on}} \equiv T^{\text{cr,n}}_{\text{on}}$.

We refer the reader to Figure 2.1 where two scenarios of jamming intervention are presented. In Scenario 1 the measurement channel is secured, while in Scenario 2 both measurement and control channel are jammed. Thus, the system dynamics (2.1) change as follows:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t)\,, \\ &u(t) = Kx(t_k)u_{\text{jmd}}(t_k)\,, \quad \forall t \in [t_k, t_{k+1}[\,, \end{aligned}$$

where operator's knowledge about the states of the plant is $x(t)$ and $x(t)u_{\text{jmd}}(t)$ for Scenarios 1 and 2, respectively.

We now consider the following problems. Let T^0 be the time difference between the initial time of the operator's clock and the DoS signal's clock, assumed to be $T^0 \ge 0$.

> *[Problem 1]*: Given a power-constrained jamming signal as in (2.2), assuming $T^n = nT$, and given $T^{\text{cr}}_{\text{off}}$, propose a time-triggered control strategy under Scenario 2 to guarantee the asymptotic stability of the system, despite lack of knowledge on T, $T^{\text{cr}}_{\text{on}}$ and the time T^0.
>
> *[Problem 2]*: Given a power-constrained jammer as in (2.2), propose (i) a time-triggered control strategy under Scenario 2, and (ii) an event-triggered control strategy under Scenario 1, to guarantee the asymptotic stability of the system, despite lack of knowledge on $\{T^n\}$, $\{T^{\text{cr,n}}_{\text{on}}\}$, $T^{\text{cr}}_{\text{off}}$, and the time, T^0.

The type of DoS signals considered here constitute a class of resource-constrained jammers, which are not necessarily malicious. Then, it is acceptable to consider

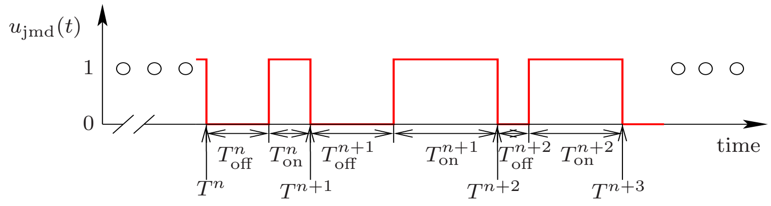


Fig. 2.2: *Scheme of the Jamming DoS Signal*

a non-malicious periodic type of disturbance as in Problem 1. The special case of Problem 1 is distinguished to show how the periodicity of the DoS signal can be exploited to limit communications over the on jamming time-intervals. Problem 2 addresses the case of non-periodic and unknown DoS signals but which are power-constrained. However, in order to deal with any signal of this class, communication over the on periods is necessary as well.

The solution of the previous problems is based on the solution to the following Intermediate Problem, which assumes further knowledge on the DoS signals:

> *[Intermediate Problem]*: Given a power-constrained jamming signal as in (2.2), knowing the sequence $\{T^n\}$ and the parameter $T^{\mathrm{cr}}_{\mathrm{off}}$, determine (i) a time-triggered control strategy under Scenario 2 in Figure 2.1b, (ii) an event-triggered control strategy under Scenario 1 in Figure 2.1a, for the system to be resilient to DoS signals.

The Intermediate Problem will be solved by concatenating time-triggered (resp. event triggered) control strategies over each subinterval $[T^n, T^{n+1}]$. More precisely, by exploiting the knowledge of $T^n, T^{\mathrm{cr}}_{\mathrm{off}}$, and T^{n+1}, the operator will increase indirectly the frequency of communication during off periods by increasing the actuation effort. Note that increasing the communication frequency alone, and keeping the actuation fixed, may not be sufficient to guarantee system stability in general. Indeed, no matter how small the initial state is, and how frequent the communication is (one can even think of a continuous-time evolution), a very active jammer can have an effect similar to an 'open-loop' regime, thus, lead to instability. We describe the overall approach in Section 3 and refer the interested reader to the corresponding stability result in Section 4.

The solution to Problem 1 consists of interspersing a learning procedure into the control strategy of the Intermediate Problem. In other words, when $T^n = nT$, and T^0 are not known, the parametric control law of the Intermediate Problem is tuned according to the most recent estimates of the parameters. Then, communications are minimally increased around the estimated on-to-off DoS times to update the estimates of the parameters. The details of the algorithm description and the correctness of the procedure are given in Subsection 5.1.

Finally, the solution to Problem 2 is also based on tuning the control law of the Intermediate Problem. However, to be able to deal with more unknown parameters, we lift the restriction of no communication during the estimated DoS periods. That is, communications according to the time or event triggered strategies are enabled also during the estimated off periods in order to estimate bounds for the unknown parameters. The details of the algorithm and stability procedure are given in Subsection 5.2.

3. Resilient Control and Triggering Strategy for the Intermediate Problem. Here, we introduce a class of control strategies for the Intermediate Problem based on a particular one-parametric family of control matrices K, along with an associated triggering time sequence, $\{t_k\}_{k\in\mathbb{N}}$. The reader is referred to the extended version of the paper online [15] for details on the proofs of all the results.

Since (A, B) is controllable, (2.1a) can be put into a controllable canonical form by a similarity transformation [2]. Thus, we focus on systems of the form:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_d & -a_{d-1} & -a_{d-2} & \cdots & -a_1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u\,,$$

$$u = [-k_d + a_d, -k_{d-1} + a_{d-1}, \cdots, -k_1 + a_1]\, x\,. \tag{3.1}$$

Moreover, the following holds.

Lemma 3.1 ([2]). *Consider $\lambda \in \mathbb{R}_{>0}$ and system (3.1). By choosing $K_\lambda = [k_1, \dots, k_d]$ as $k_i = \frac{d!}{i!(d-i)!}\lambda^i, \quad i \in \{1, \dots, d\}$, all the closed-loop system poles are placed at $-\lambda$. The eigenvalue $-\lambda$ has algebraic multiplicity d and geometric multiplicity 1.* •

Remark 3.2. *Note that matrix $A + BK_\lambda$ has only one linearly independent eigenvector, therefore it is not diagonalizable. This property holds for all values of $\lambda \in \mathbb{R}_{>0}$. Moreover, let v be an eigenvector of $A + BK_\lambda$. Then, since the matrix $A + BK_\lambda + \lambda I$ depends on λ in a polynomial way, the components of this eigenvector, given by $(A + BK_\lambda + \lambda I)v = 0$, become rational functions of λ.*

From Lemma 3.1, the Jordan decomposition of this matrix can be expressed as:

$$A + BK_\lambda = T_\lambda J_\lambda T_\lambda^{-1}\,, \tag{3.2}$$

where $J_\lambda = -\lambda I + N$ and T_λ is a matrix composed of linearly independent and generalized eigenvectors. The matrix N has a *unique* structure for all values of λ, as the geometric multiplicity of this eigenvalue remains unchanged. Moreover, by Remark 3.2 and the construction of the generalized eigenvectors [10], the entries of T_λ and T_λ^{-1} are rational functions of λ.

Consider the system (3.1) with $u(t) = K_\lambda x(t_k)$. Then, the closed-loop dynamics is:

$$\dot{x} = (A + BK_\lambda)x + BK_\lambda e\,,$$

where $e(t) = x(t_k) - x(t)$. Applying the static transformations $e = T_\lambda e_\lambda$, and $x = T_\lambda x_\lambda$, we obtain:

$$\dot{x}_\lambda = J_\lambda x_\lambda + T_\lambda^{-1} BK_\lambda T_\lambda e_\lambda\,. \tag{3.3}$$

Based on the previous Jordan decomposition technique and the unique structure of N, we are able to find a *common Lyapunov function* for sufficiently large λ that will help in the design our triggering strategies later on. The ISS-based triggering approach developed in papers [28] and [22] have inspired the derivation of this result.

Lemma 3.3. *Take $\lambda > \|N\| + 1/2$, and K_λ as in Lemma 3.1. Then $V(x_\lambda) = x_\lambda^T x_\lambda$ is a common ISS-Lyapunov function for the system (3.3), and the event-triggered* *condition:*

$$|e_\lambda(t)|^2 \leq \frac{\sigma(2\lambda - 1 - 2\|N\|)}{\|T_\lambda^{-1} BK_\lambda T_\lambda\|^2} |x_\lambda(t)|^2 \,, \tag{3.4}$$

guarantees the asymptotic stability of the system, for $\sigma \in (0, 1)$. Accordingly, the associated triggering time-sequence, $\{t_k\}_{k\in\mathbb{N}}$, is generated as follows:

$$t_{k+1} = \inf\left\{ t > t_k \middle| |e_\lambda(t)|^2 \geq \frac{\sigma(2\lambda - 1 - 2\|N\|)}{\|T_\lambda^{-1} BK_\lambda T_\lambda\|^2} |x_\lambda(t)|^2 \right\}, \quad k \in \mathbb{N} \,. \tag{3.5}$$

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Remark 3.4. *Let t_k and t_{k+1} be two consecutive time-instants given by the event-triggering strategy* (3.5)*. Then, for each λ, the following holds:*

$$\exists \text{ a largest } \tau_\lambda > 0, \text{ such that } t_{k+1} - t_k \geq \tau_\lambda, \forall k \in \mathbb{N} \,.$$

That is, the parameter τ_λ is the largest uniform lower-bound for the triggering time-sequence, $\{t_k\}$ given by (3.5)*. Theorem III.1 presented in [28], shows how to compute such largest lower bound τ_λ, which is recalled in Algorithm 1 of Section 6. This also implies that the time-sequence, $\{t_k\}$, generated by* (3.5) *does not accumulate; that is, for a fixed λ, we have $\lim_{k\to\infty} t_{k+1} - t_k \neq 0$. Since under Scenario 2 we do not assume that the operator can continuously measure the plant states, we will adopt this τ_λ as the basis of our economic time-triggered control strategy.*

For the parameter τ_λ, and sequence $\{t_k\}_{k\in\mathbb{N}}$, we show the following property.

Proposition 3.5. *Let $\lambda > \|N\| + 1/2$, and let $\{t_k\}_{k\in\mathbb{N}}$ be the associated time-sequence generated by the event-triggering strategy* (3.5)*. Consider the parameter τ_λ introduced in Remark 3.4. Then, the following holds:*

$$\lim_{\lambda\to\infty} \tau_\lambda = 0 \,, \quad \text{and} \quad \lim_{\lambda\to\infty} t_{k+1} - t_k = 0 \,, \; \forall k \in \mathbb{N} \,. \tag{3.6}$$

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At this point, we present the class of triggering strategies we consider to solve the Intermediate Problem (both scenarios) starting at T^0. The idea of our approach is the following. It is clear that, by from Proposition 3.5, the communication times will be reduced by increasing λ.

The stability characterization using these strategies is postponed to Section 4. To do this, we consider the jammer is constantly maintaining a *"worst-case jamming scenario,"* i.e., $T^n_{\text{off}} = T^{\text{cr}}_{\text{off}}$, $\forall n \in \mathbb{Z}$. We would like to clarify that this is a worst case, because the jammer is active the most and is inactive the least, i.e., T^n_{off} takes its least value for each jamming time interval.

Definition 3.6. *A time-triggered control strategy for the Intermediate Problem, Scenario 2, consists of $u_n(t) = K_{\lambda_n} x(t^*_{k,n})$ during $t \in [t^*_{k,n}, t^*_{k+1,n}[$, $k, n \in \mathbb{N}$, where the $t^*_{k,n}$ are the time instants:*

$$t^*_{k,n} \in \{l\tau_{\lambda_n} \mid l\tau_{\lambda_n} \in [T^{n-1}, T^{n-1} + T^{\text{cr}}_{\text{off}}], l \in \mathbb{N}\} \cup \{T^n\} \,, \tag{3.7}$$

Here, K_{λ_n} is chosen according to Lemma 3.1 and λ_n so that $\tau_{\lambda_n} \in [T^{n-1}, T^{n-1} + T^{\text{cr}}_{\text{off}}]$, for all $n \in \mathbb{N}$, which is guaranteed by Proposition 3.5.

Note that in the previous definition, the particular λ used over different time periods $[T^{n-1}, T^{n-1}+T_{\rm off}^{\rm cr}]$, $n \in \mathbb{N}$, can change from period to period, and this is denoted by λ_n. The chosen λ_n determines both the sequence of communication times $\{t^*_{k,n}\}$ as well as the control matrix K_{λ_n} used for feedback control. We also note that, based on Proposition 3.5, and for a given n, one can find a λ_c so that the multiples of τ_λ lie in the desired interval, for $\lambda \geq \lambda_c$, i.e., the set of $l\tau_{\lambda_n}$ introduced in (3.7) is never empty for a sufficiently large λ. Finally, note that these strategies limit communications to the off periods of the jamming signal.

Definition 3.7. *An event-triggered control strategy for the Intermediate Problem, Scenario 1 consists of $u_n(t) = K_{\lambda_n} x(t^*_{k,n})$ during $t \in [t^*_{k,n}, t^*_{k+1,n}[$, $k, n \in \mathbb{N}$, where the $t^*_{k,n}$ are the time instants:*

$$t^*_{k,n} \in \{t_l \textit{ satisfying } (3.5) \mid t_l \in [T^{n-1}, T^{n-1}+T_{\rm off}^{\rm cr}], l \in \mathbb{N}\} \cup \{T^n\}\,. \tag{3.8}$$

Here, K_{λ_n} is chosen according to Lemma 3.1 and λ_n so that the first event-triggered instant, t_1, satisfies $t_1 \in [T^{n-1}, T^{n-1}+T_{\rm off}^{\rm cr}]$, for all $n \in \mathbb{N}$, which is guaranteed by Proposition 3.5.

Similar remarks apply here as in after Definition 3.6. We would like to emphasize that according to (3.8), t_l are just those time instants declared by (3.5), which also lie in the desired interval, $[T^{n-1}, T^{n-1}+T_{\rm off}^{\rm cr}]$.

The choice of λ_n, for each $n \in \mathbb{N}$, which influences both the control effort, K_{λ_n}, and the frequency of communications, will be made specific in the following section. We note that both effort K_{λ_n} and frequency of communications will be used to guarantee asymptotic stability of the linear system.

4. Stability Analysis for the Intermediate Problem. In this section, we present a proof statement of how the class of control and triggering strategies discussed in earlier sections are able to solve the Intermediate Problem (both scenarios) for an appropriate choice of λ_n. The analysis included in [15] provides the foundation to solve Problems 1, and 2, and hence to deal with unknown DoS signals.

Given $M \in \mathbb{R}^{d\times d}$, define the μ operator:

$$\mu(M) = \max\left\{\mu \mid \mu \in \operatorname{spec}\left(\frac{M+M^T}{2}\right)\right\}, \tag{4.1}$$

with spec(.) be the set of eigenvalues. Moreover, we would like to recall the following lemma.

Lemma 4.1 ([21]). *Consider the polynomial:*

$$p(z) = a_0 + a_1 z + \cdots + a_d z^d\,, \tag{4.2}$$

where, $z \in \mathbb{R}$, and $a_i \in \mathbb{R}$, for $i \in \{1, \dots, d\}$. Then, a lower-bound for all the roots of $p(z) = 0$ is given as follows:

$$R = \frac{|a_0|}{\max\left(|a_0|, |a_1| + |a_2| + \cdots + |a_d|\right)}\,. \tag{4.3}$$

We can now state the main result of this section.

Theorem 4.2. *(Stability Characterization of Intermediate Problem, Scenario 2): Consider System* (3.1)*, where* (A, B) *is a controllable pair. Given a jamming signal* (2.2)*, where the sequence* $\{T^n\}$ *and parameter* $T_{\text{off}}^{\text{cr}}$ *are known; consider:*

$$C(n,\lambda) \triangleq \left(\frac{\exp\left(-(1-\sigma)(2\lambda-1-2\|N\|)T_{\text{off}}^{\text{cr}}/4\right)}{\|T_\lambda^{-1}\|^{-1}\sqrt{R_\lambda}}\right) \times \left(\frac{\|BK_\lambda\|}{\mu_A}(\exp(T_{\text{on}}^{\text{cr,n}}\mu_A)-1) + \frac{\exp\left(-(1-\sigma)(2\lambda-1-2\|N\|)\tau_\lambda\right)}{\|T_\lambda^{-1}\|^{-1}\sqrt{R_\lambda}}\exp(T_{\text{on}}^{\text{cr,n}}\mu_A)\right), \tag{4.4}$$

wherein R_λ *is as defined in* (4.3) *for the characteristic polynomial of the matrix,* $(T_\lambda^{-1})^T(T_\lambda^{-1})$*, and* $\sigma \in (0,1)$*. Let* $\lambda_n^* = \inf\{\lambda_n | C(n,\lambda_n) < 1 and\, \lambda_n > \|N\| + 1/2\}$*, then, for each* $n \in \mathbb{N}$*, applying* K_{λ_n} *as in Lemma 3.1, along with the time-triggered strategy* (3.7)*, for any* $\lambda_n \geq \lambda_n^*$*, renders the system asymptotically stable.* •

Theorem 4.2 is based on the class of time-triggered strategies stated in Definition 3.6. The following corollary characterizes the alternative class of event-triggered strategy of Definition 3.7 to solve the Intermediate Problem, Scenario 1.

Corollary 4.3. *(Stability Characterization of the Intermediate Problem, Scenario 1): Consider System* (3.1)*, where* (A, B) *is a controllable pair. Given a jamming signal* (2.2)*, where the sequence* $\{T^n\}$ *and parameter* $T_{\text{off}}^{\text{cr}}$ *are known; recall then* $C(n, \lambda)$ *as characterized in* (4.4) *of Theorem 4.2 and let* $\lambda_n^* = \inf\{\lambda_n | C(n, \lambda_n) < 1 \, and, \lambda_n > \|N\| + 1/2\}$*. Then, for each* $n \in \mathbb{N}$*, applying* K_{λ_n} *as in Lemma 3.1, along with the event-triggered strategy* (3.8)*, for any* $\lambda_n \geq \lambda_n^*$*, renders the system asymptotically stable.* •

Remark 4.4. *We would like to emphasize that in our proposed solutions to the Intermediate Problem (both scenarios) and in order to deal with a power-constrained DoS jamming signal, the operator chooses a parameter,* λ*, affecting the "frequency of communication," characterized by* τ_λ*, indirectly through the "actuation effort," characterized by* K_λ*. In this setting, our objectives are: (i) to determine theoretically how this strategy of tuning* λ *can work, and, (ii) to find a least value for* λ *such that for a given* $T_{\text{off}}^{\text{cr}}$*,* $\{T^n\}$*, we can still guarantee the stability of the system with the associated largest period* τ_λ*.*

5. Joint Triggering Control and Jammer Identification for the Solution to Problems 2 and 3. In this section, we propose our solutions to Problems 2 and 3, which are built on the resilient control and triggering strategies introduced in Section 3 and analyzed in Section 4. First we discuss the JAMCOID for Periodic Signals algorithm to solve Problem 1. Then, based on the obtained observations we develop the JAMCOID algorithm to solve Problem 2. See [15] for more information.

First, let us denote by $u_{\text{id}} : \mathbb{R}_{\geq 0} \to \{\texttt{null}\} \cup \{1\}$, the signal that the operator uses for jammer identification purposes, where $u_{\text{id}}(t) = 1$ means that the operator sends message 1 to the plant at time t, and $u_{\text{id}}(t) = \texttt{null}$ represents that no message is submitted. Let us also denote by $u_{\text{ste}} : \mathbb{R}_{\geq 0} \to \{\texttt{null}\} \cup \mathbb{R}^d$ the signal rebound from the plant, such that $u_{\text{ste}}(t) \in \mathbb{R}^d$ contains a successfully delivered message containing state-update information at time t, whereas $u_{\text{ste}}(t) = \texttt{null}$ represents no message is delivered at the operator's side. Finally, let $u_{\text{ctrl}} : \mathbb{R}_{\geq 0} \to \{\texttt{null}\} \cup \mathbb{R}$ be the submitted control, where similar to the u_{id}-case, $u_{\text{ctrl}}(t) \neq \texttt{null}$ induces that a control $u_{\text{ctrl}}(t)$ is computed and sent, whereas $u_{\text{ctrl}}(t) = \texttt{null}$ means that no message is sent.

We assume that, from the operator viewpoint, the submission of u_{id}, receipt of u_{ste}, and submission of u_{ctrl} happen in a sequential and instantaneous manner, respectively. Note that $u_{\mathrm{ctrl}}(t) = \texttt{null}$ if $u_{\mathrm{ste}}(t) = \texttt{null}$, i.e., we do *not* send any control if we do *not* receive any measurement, and this happens when the jammer is active at t.

Finally, we introduce various symbols and parameters used in them. The parameter, T_j^0 is the time difference between the jammer's and operator's clocks at the j-th iteration of the algorithm—with $T^0 \equiv T_0^0$. The parameter M is the sampling time with which the operator communicates with the plant. The parameters, $\hat{T}_{\mathrm{off}}^{\mathrm{cr}}$ and $\hat{T}_{\mathrm{on}}^{\mathrm{cr}}$ are, respectively, the estimate of $T_{\mathrm{off}}^{\mathrm{cr}}$ and $T_{\mathrm{on}}^{\mathrm{cr}}$. The parameter, $\widehat{lT^{\mathrm{next}}}$ is the estimate of the l-th multiple of the period T that is used in JAMCOID for Periodic Signals algorithm. The parameter, $\sigma \in (0, 1)$ is also used in order to refine the sampling time, M, if required.

5.1. The JAMCOID for Periodic Signals Algorithm.

Unlike in Section 3, we assume here that the operator's and jammer's clocks do not have to be synchronous but have similar linear models. Let T^0 be the time difference between the jammer's clock initial time and the operator's. W.l.o.g. assume $T^0 \geq 0$. We realize that, under the *"worst-case jamming scenario,"* there are three unknown parameters, $T_{\mathrm{on}}^{\mathrm{cr}}$, T and T^0, which characterize the jammer's signal together with the known parameter, $T_{\mathrm{off}}^{\mathrm{cr}}$.

Intuitively, the core idea behind JAMCOID for Periodic Signals is to intelligently generate the triggering time-sequence $\{t_k\}$ in order to, (i) bound the asynchronicity, T^0, (ii) find a valid useful interval to which the parameter T, or some multiple of it, belongs. In this way, we can reliably estimate the times when the signal will go from on to off, which will allows us to implement the economic strategy of the previous section (restricting communications to the non-active jammer periods) and guarantee stability. The algorithm is described in the flowchart of Figure 5.1, where $u_{\mathrm{ctrl}}(t_k)$ is computed as explained in Section 3, that is $u_{\mathrm{ctrl}}(t_k) = K_\lambda u_{\mathrm{ste}}(t_k)$, with the gains K_λ given in Lemma 3.1. In the flowchart of Figure 5.1: (i) *law 0* refers to a periodic time-triggered strategy defined by the period M, and with associated K_λ; (ii) *law 1* refers to a time-triggered strategy of the class in Definition 3.6 with a λ chosen to guarantee the conditions of Theorem 4.2 for an estimated off period of $\hat{T}_{\mathrm{off}}^{\mathrm{cr}}$ and on period $\hat{T}_{\mathrm{on}}^{\mathrm{cr}}$.

Following along the flowchart, the next steps are taken:

Step I: The operator starts using a periodic communication and control time-triggered strategy, *law* 0, with an initial $\tau_{\lambda_0} = M < \frac{T_{\mathrm{off}}^{\mathrm{cr}}}{2}$ that should be also sufficient to stabilize the system.

With this, the operator verifies the success or failure of the transmitted signal at each sampling time. One can then distinguish two cases:

Case (1): Transmissions *never* hit the jammer's on-subperiod, that is, $u_{\mathrm{ste}}(t_k) \neq \texttt{null}$, $\forall\, t_k$. Thus, we can keep updating our control at the prescribed time-instants, $t_k = kM$, without interruption. This can happen if, in fact, there is no jammer or, in case there is, the clocks are synchronized and the jamming on-subperiod falls between consecutive triggering time-instants.

Case (2): We detect a first on-to-off jamming signal transition. That is:

$$\exists k_1 \text{ such that} \quad u_{\mathrm{ste}}(k_1 M) = \texttt{null} \quad \text{and} \quad u_{\mathrm{ste}}((k_1+1)M) \neq \texttt{null}\,,$$

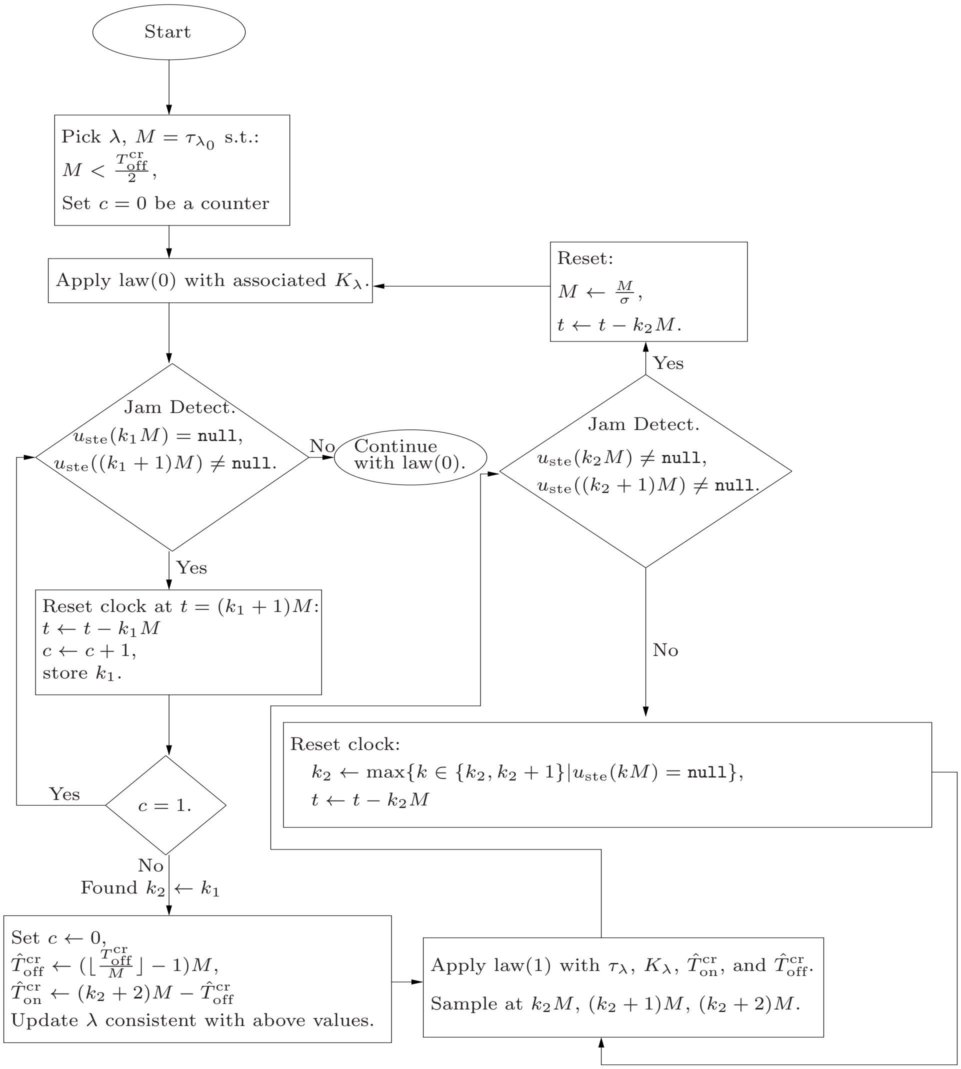
Start
Pick λ, $M = \tau_{\lambda_0}$ s.t.:
$M < \frac{T_{\text{off}}^{\text{cr}}}{2}$,
Set $c = 0$ be a counter
Apply law(0) with associated K_λ.
Jam Detect.
$u_{\text{ste}}(k_1 M) = \texttt{null}$,
$u_{\text{ste}}((k_1+1)M) \neq \texttt{null}$.
No
Continue with law(0).
Yes
Reset clock at $t = (k_1+1)M$:
$t \leftarrow t - k_1 M$
$c \leftarrow c + 1$,
store k_1.
Yes
$c = 1$.
No
Found $k_2 \leftarrow k_1$
Set $c \leftarrow 0$,
$\hat{T}_{\text{off}}^{\text{cr}} \leftarrow (\lfloor \frac{T_{\text{off}}^{\text{cr}}}{M} \rfloor - 1)M$,
$\hat{T}_{\text{on}}^{\text{cr}} \leftarrow (k_2+2)M - \hat{T}_{\text{off}}^{\text{cr}}$
Update λ consistent with above values.
Apply law(1) with τ_λ, K_λ, $\hat{T}_{\text{on}}^{\text{cr}}$, and $\hat{T}_{\text{off}}^{\text{cr}}$.
Sample at $k_2 M$, $(k_2+1)M$, $(k_2+2)M$.
Reset:
$M \leftarrow \frac{M}{\sigma}$,
$t \leftarrow t - k_2 M$.
Yes
Jam Detect.
$u_{\text{ste}}(k_2 M) \neq \texttt{null}$,
$u_{\text{ste}}((k_2+1)M) \neq \texttt{null}$.
No
Reset clock:
$k_2 \leftarrow \max\{k \in \{k_2, k_2+1\} | u_{\text{ste}}(kM) = \texttt{null}\}$,
$t \leftarrow t - k_2 M$

Fig. 5.1: *Flowchart of* JAMCOID FOR PERIODIC SIGNALS *Algorithm*

where recalling the jamming signal shape, the following holds:

$$\exists k_1 \text{ and } l_1 \text{ such that} \quad k_1 M < T_1^0 + l_1 T \leq (k_1+1)M\,. \tag{5.1}$$

The first on-to-off switch time k_1 is stored, and $c = 1$ represents in the flowchart that a first on-to-off switch was detected.

Step II: After detecting the jammer is on, the operator applies a first clock reset so that the clock time difference is upper bounded by the sampling time, M. In formal words, at $t = (k_1+1)M$, we reset $t \leftarrow t - k_1 M$. Then, the counter c is reset to 2. Let us denote $T_2^0 = T_1^0 + l_1 T - k_1 M$, then by (5.1) it holds that:

$$0 < T_2^0 \leq M\,. \tag{5.2}$$

Step III: The operator repeats the strategy described in **Step I** to obtain an estimate of the time between two consecutive on-to-off switches. In other words, this would give a rough estimate of a multiple of T, which will be used later to limit communications. Again, two cases are possible:

Case (1): We do *never* hit the jammer's on-subperiod, that is $u_{\text{ste}}(t_k) \neq \texttt{null}, \forall\, t_k$. In this case, we keep updating our control at the prescribed times without interruption. As before, this can be due to a synchronization with the DoS signal and because the on-subperiods fall between consecutive triggering time-instants.

Case (2): We detect a second on-to-off signal transition. Let $k_2 \in \mathbb{N}$ such that:

$$\exists k_2 \,\text{such that} \quad u_{\text{ste}}(k_2 M) = \texttt{null} \quad \text{and} \quad u_{\text{ste}}((k_2+1)M) \neq \texttt{null}\,,$$

where recalling the jamming signal shape, the following holds:

$$\exists k_2 \,\text{and}\, l_2 \,\text{such that} \quad k_2 M < T_2^0 + l_2 T \leq (k_2+1)M\,. \tag{5.3}$$

Step IV: A new clock reset is applied, to maintain the time offset bounded by the sampling time M. Using the newly identified k_2, the operator can now find bounds when the DoS signal will switch from on to off again.

This will be helpful to limit the amount of communications used to probe the DoS signal. In formal words, at time-instant, $t = (k_2+1)M$, the operator resets the clock as $t \leftarrow t - k_2 M$. Further, denote $T_3^0 = T_2^0 + l_2 T - k_2 M$, according to Equation (5.3), we get:

$$0 < T_3^0 \leq M\,, \tag{5.4}$$

where, additionally, it can be proven that:

$$(k_2-1)M < T_3^0 + l_2 T \leq (k_2+2)M\,. \tag{5.5}$$

The time $T_3^0 + l_2 T$ is an unknown time that represents a DoS signal on to off transition. By the above, we know this is located within $[(k_2-1)M, (k_2+2)M]$.

Step V: The control law will now be changed to a resilient strategy with proper choice of λ, *law* 1. Let $s = \lfloor \frac{T_{\text{off}}^{\text{cr}}}{M} \rfloor$ and consider the time-interval $[M, sM]$. Since $0 < T_3^0 \leq M$, from definition of s, $sM \leq T_{\text{off}}^{\text{cr}}$ follows. Also, communication with the plant is feasible at any time in $[M, sM]$. Hence, $[M, sM]$ can play the role of $[0, T_{\text{off}}^{\text{cr}}]$ in the known jammer scenario. From (5.5), note that $(k_2+2)M$ is a valid upper-bound for the unknown parameter $T_3^0 + l_2 T$. Thus, we estimate $l_2 T$ by $(k_2+2)M$. In addition, provided these information, we compute $\hat{T}_{\text{off}}^{\text{cr}} = (s-1)M$, $\hat{T}_{\text{on}}^{\text{cr}} = (k_2+2)M - (s-1)M$. Since $k_2 \geq s+1$, then we have that $\hat{T}_{\text{on}}^{\text{cr}} \geq M$. We then plug these parameters back into Equation (4.4) for $C(n,\lambda)$, and retrieve the proper λ^* for which $C(n,\lambda^*) < 1$; accordingly, $K_\lambda \leftarrow K_{\lambda^*}$ and $\tau_\lambda \leftarrow \tau_{\lambda^*}$. We then keep updating the control as $u_{\text{ctrl}}(t_k) = K_\lambda x(t_k)$, where

$$t_k \in \big\{ l\tau_\lambda \mid l\tau_\lambda \in [M, sM] \big\} \cup \big\{ (k_2+2)M \big\}\,, \quad \forall \lambda \in \mathbb{R}_{>0}\,. \tag{5.6}$$

In addition to communicating with the plant at the time-instants in (5.6), the operator will also send messages at times $t \in \{k_2 M, (k_2+1)M\}$. The following may occur:

Case (1): It holds that $u_{\text{ste}}(k_2 M) \neq \texttt{null} \neq u_{\text{ste}}((k_2+1)M)$. This means the operator does not detect the on-to-off transition in the new estimated period, and, since it had previously detected a null message at k_2M and a non-null message at $(k_2+1)M$, it must be that either the length of the on subperiod is shorter than M (if it does fall between k_2M and $(k_2+1)M$) or it ends before k_2M. Therefore, in this case, we reset $M \leftarrow \frac{M}{\sigma'}$, where $\sigma' \in (1,\infty)$ is a design parameter. We then reset $t \leftarrow t - k_2M$ and repeat from **Step I**.

Case (2): Either $u_{\text{ste}}(k_2M) = \texttt{null}$, or $u_{\text{ste}}((k_2+1)M) = \texttt{null}$, or both. This is characterized by $\bar{k}M$, where:

$$\bar{k} = \max\{k \in \{k_2, k_2+1\} \big| u_{\text{ste}}(kM) = \texttt{null}\}\,.$$

Reset $k_2 \leftarrow \bar{k}$, $t \leftarrow t - \bar{k}M$, and $T_3^0 \leftarrow T_3^0 + l_2T - \bar{k}M$, for which (5.5) also holds. Repeat from **Step V**.

The system asymptotic stability employing JAMCOID for Periodic Signals, is shown in the next theorem.

Theorem 5.1. *Consider System* (3.1)*, where* (A,B) *is a controllable pair, and a jamming signal* (2.2) *with constant unknown parameters,* T*,* $T_{\text{off}}^{\text{cr}}$*,* $T_{\text{on}}^{\text{cr}}$*, and constant known parameter,* $T_{\text{off}}^{\text{cr}}$*. The algorithm* JAMCOID for Periodic Signals *renders the system asymptotically stable.* •

5.2. The JAMCOID Algorithm. In order to solve Problem 2, we present here the JAMCOID algorithm. The main idea behind JAMCOID is to generate the triggering time-sequence $\{t_k\}$ in order to (i) bound the asynchronicity by applying appropriate clock resets, and (ii) find an underestimate of $T_{\text{off}}^{\text{cr}}$ and an overestimate of all $\{T_{\text{on}}^{\text{cr,n}}\} < \infty$. Intuitively, JAMCOID employs a time-triggered strategy which is adapted after the estimates of the off and on jamming periods are refined as the algorithm is run online. As in the previous JAMCOID for Periodic Signals, the asynchronicity between jammer's and operator's clocks is also bounded by reseting the operator's clock once both estimates have been retrieved. The difference between JAMCOID for Periodic Signals and JAMCOID is that, in the latter, the structure of the jamming signal is exploited in order to limit communications as much as possible during the viable times. However, in JAMCOID communications are not interrupted in order to learn and update the uniform lower underestimates of the off period and an overestimate of the on periods. Therefore, this will lead to a conservative algorithm that can handle any power-constrained signal.

As mentioned earlier, JAMCOID provides a solution based on a time-triggered control strategy under Scenario 2; nonetheless, we shall discuss an extension to an event-triggering control strategy. The algorithm is described in the flowchart of Figure 5.2. In this flowchart, while following the intuitive explanation stated earlier, *law 0* refers to a periodic time-triggered strategy defined by the period $M = \tau_\lambda$, and with associated K_λ. Briefly, the following steps are performed:

Step I: The operator sends messages to the plant with control content following a periodic triggering strategy. During this phase, we can distinguish between two cases.

Case (1): We do *never* hit the jammer's on-subperiod, that is, $u_{\text{ste}}(t_k) \neq \texttt{null}$, $\forall\, t_k$. Thus, we can keep updating our control at the prescribed time-instants, $t_k = kM$, without interruption.

Case (2): We detect an on-to-off jamming signal transition. That is first we detect:

$$\exists k_1 \text{ such that} \quad u_{\text{ste}}(k_1M) \neq \texttt{null} \quad \text{and} \quad u_{\text{ste}}((k_1+1)M) = \texttt{null}\,,$$

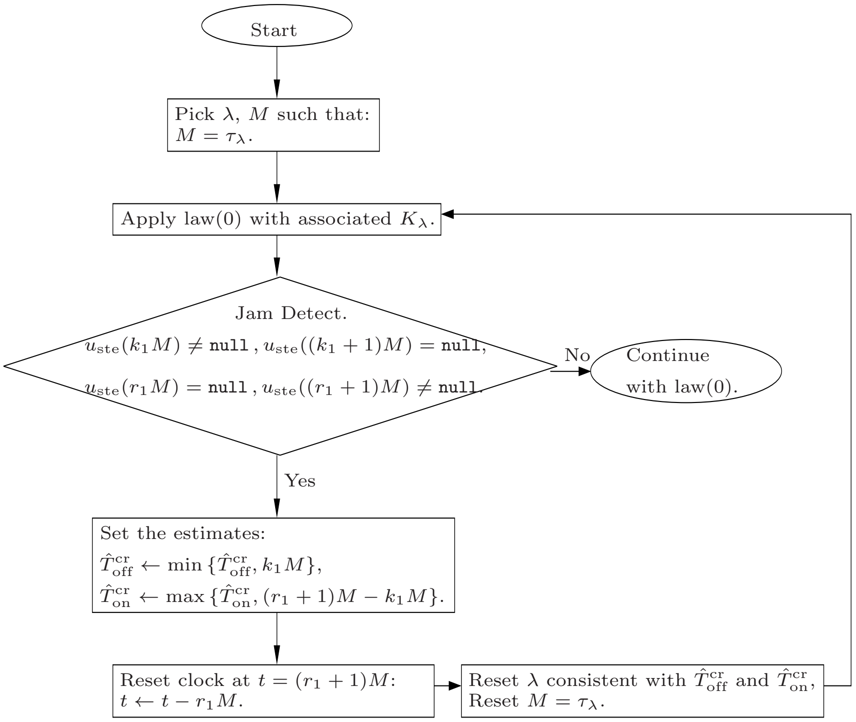
Start
Pick λ, M such that:
$M = \tau_\lambda$.
Apply law(0) with associated K_λ.
Jam Detect.
$u_{\rm ste}(k_1 M) \neq \texttt{null}\,, u_{\rm ste}((k_1+1)M) = \texttt{null}$,
$u_{\rm ste}(r_1 M) = \texttt{null}\,, u_{\rm ste}((r_1+1)M) \neq \texttt{null}$.
No
Continue
with law(0).
Yes
Set the estimates:
$\hat{T}^{\rm cr}_{\rm off} \leftarrow \min\{\hat{T}^{\rm cr}_{\rm off}, k_1 M\}$,
$\hat{T}^{\rm cr}_{\rm on} \leftarrow \max\{\hat{T}^{\rm cr}_{\rm on}, (r_1+1)M - k_1 M\}$.
Reset clock at $t = (r_1+1)M$:
$t \leftarrow t - r_1 M$.
Reset λ consistent with $\hat{T}^{\rm cr}_{\rm off}$ and $\hat{T}^{\rm cr}_{\rm on}$,
Reset $M = \tau_\lambda$.

Fig. 5.2: *Flowchart of* JAMCOID *Algorithm*

and then we detect,

$$\exists r_1 \,\text{such that} \quad u_{\rm ste}(r_1 M) = \texttt{null} \quad \text{and} \quad u_{\rm ste}((r_1+1)M) \neq \texttt{null}\,,$$

where recalling the jamming signal shape, the following holds:

$$\exists r_1 \,\text{and}\, s_1 \,\text{such that} \quad r_1 M < T_1^0 + T^{s_1} \leq (r_1+1)M\,. \tag{5.7}$$

In addition, the following estimates can be obtained:

$$\hat{T}^{\rm cr,1}_{\rm off} = k_1 M\,, \quad \text{and}\,, \quad \hat{T}^{\rm cr,1}_{\rm on} = (r_1+1)M - k_1 M\,.$$

Step II: After detecting the jammer is on, the operator applies a first clock reset so that the clock time difference is upper bounded by the sampling time, M. That is, at $t = (r_1+1)M$, the clock is reset as $t \leftarrow t - r_1 M$. Let us denote $T_2^0 = T_1^0 + T^{l_1} - r_1 M$, then by (5.7) it holds that $0 < T_2^0 \leq M$. In addition, by obtaining the estimates, $\hat{T}^{\rm cr,1}_{\rm off}$ and $\hat{T}^{\rm cr,1}_{\rm on}$, we shall find the minimum off-subperiod and maximum on-subperiod that is computed up to this stage of the algorithm. In other words:

$$\hat{T}^{\rm cr}_{\rm off} \leftarrow \min\{\hat{T}^{\rm cr,1}_{\rm off}, \hat{T}^{\rm cr}_{\rm off}\}\,, \quad \text{and}\,, \quad \hat{T}^{\rm cr}_{\rm on} \leftarrow \max\{\hat{T}^{\rm cr,1}_{\rm on}, \hat{T}^{\rm cr}_{\rm on}\}\,.$$

Once we have found the estimates $\hat{T}^{\rm cr}_{\rm off}$ and $\hat{T}^{\rm cr}_{\rm on}$, we plug the different parameters back into (4.4) for $C(n,\lambda)$, and retrieve the proper λ^* for which $C(n,\lambda^*) < 1$. We then update $\tau_\lambda \leftarrow \tau_{\lambda^*}$, $K_\lambda \leftarrow K_{\lambda^*}$, and go back to **Step I**.

The asymptotic stability of the system, employing JAMCOID, is stated next.

Theorem 5.2. *Consider System* (3.1)*, where* (A, B) *is a controllable pair, and a general jamming signal* (2.2) *with unknown parameters,* $\{T^n\}$*,* $\{T_{\text{on}}^{\text{cr,n}}\}$*, and* $T_{\text{off}}^{\text{cr}}$*. The algorithm* JAMCOID *renders the system asymptotically stable.* •

The JAMCOID algorithm is based on a time-triggered control strategy, which would be a solution under Scenario 2. In the following, we discuss an adaptation of this algorithm to deal with an event-triggered control strategy.

Remark 5.3. *The* JAMCOID *algorithm can be adapted for event-triggered strategies, which then provides a solution under Scenario 1. Let* $\{t_k\}$ *be the event-triggered condition as described in Equation* (3.5)*, Lemma 3.3. Then, the* JAMCOID *proposed for time-triggered strategies can be changed as follows:*

1. *In* ***Step I–Case (2)****, we shall first detect:*

$$\exists k_1 \textit{ such that } \quad u_{\text{ste}}(t_{k_1}) \neq \boldsymbol{null} \quad \textit{and} \quad u_{\text{ste}}(t_{k_1+1}) = \boldsymbol{null}\,,$$

   and then we detect,

$$\exists r_1 \textit{ such that } \quad u_{\text{ste}}(t_{r_1}) = \boldsymbol{null} \quad \textit{and} \quad u_{\text{ste}}(t_{r_1+1}) \neq \boldsymbol{null}\,,$$

   which then implies the following:

$$\exists r_1 \textit{ and } s_1 \textit{ such that } \quad t_{r_1} < T_1^0 + T^{s_1} \leq t_{r_1+1}\,,$$

   which is a counterpart to (5.7)*. This then provides the estimates,* $\hat{T}_{\text{off}}^{\text{cr},1} = t_{k_1}$*, and,* $\hat{T}_{\text{on}}^{\text{cr},1} = t_{r_1+1} - t_{k_1}$*.*
2. *In* ***Step II****, the reset is performed at* $t = t_{r_1+1}$ *as* $t \leftarrow t - t_{r_1}$*, whereby the estimates are updated as* $\hat{T}_{\text{off}}^{\text{cr}} \leftarrow \min\{\hat{T}_{\text{off}}^{\text{cr},1}, \hat{T}_{\text{off}}^{\text{cr}}\}$*, and,* $\hat{T}_{\text{on}}^{\text{cr}} \leftarrow \max\{\hat{T}_{\text{on}}^{\text{cr},1}, \hat{T}_{\text{on}}^{\text{cr}}\}$*. Then, proper* λ^* *shall be obtained by resorting to* (4.4)*, by means of which the update on* $K_\lambda \leftarrow K_{\lambda^*}$*, and event-triggered condition* (3.5) *shall be performed.*

In effect, the proof of this extension can be performed in an exact similar way as in proof of Theorem 5.2—this time by resorting to Corollary 4.3.

Remark 5.4. *The major difference between the* JAMCOID *algorithm and the* JAMCOID for Periodic Signals *algorithm is that the latter is more economic in terms of communications than the former. In other words, the periodicity property, along with the knowledge of* $T_{\text{off}}^{\text{cr}}$ *in* JAMCOID for Periodic Signals*, lets us first identify the time-intervals where communications are guaranteed and hence develop a triggering strategy to ensure the stability. Nevertheless, in* JAMCOID*, because of the non-periodicity of the jamming signal, which prohibits possible predictions on the on-to-off transition time-instants, and lack of knowledge on* $T_{\text{off}}^{\text{cr}}$*, we have to always communicate over the active jamming time-intervals, as well, in order to update our estimates on the minimum of* $T_{\text{off}}^{\text{cr}}$ *and* $\max_{n\in\mathbb{N}}\{T_{\text{on}}^{\text{cr,n}}\}$ *to ensure the system stability. This, hence, prevents a more economic number of communications.*

6. Simulations. Having established theoretical results in previous sections, here we demonstrate their functionality on an academic example. Hence, we break this section into two parts; first we discuss the known jammer scenario, followed by the unknown jammer scenario.

Algorithm 1 $C(\lambda)$-Seeking

```
Input: Matrices: $A$, $B$, and $N$, Sequence: $\{\lambda_k\}_{k=1}^{N'}$, Parameters: $\sigma$, $T_{\text{off}}^{\text{cr}}$, and $T$.
Given controllable pair $(A, B)$, compute the proper similarity transformation matrix, and find $(A_c, B_c)$—which are in controllable canonical form,
for $k = 1$ to $N'$ do
    Numerically solve the following ODE, with $\phi(0) = 0$:
        $\dot{\phi} = \|A + BK_{\lambda_k}\| + (\|A + BK_{\lambda_k}\| + \|BK_{\lambda_k}\|)\phi + \|BK_{\lambda_k}\|\phi^2$ ,
    Find $\tau_{\lambda_k}$, such that $\phi(\tau_{\lambda_k}) = \sigma$,
    Compute $C(\lambda_k) \equiv C(1, \lambda_k)$, as stated in (4.4) for $n = 1$,
end for
Output: Sequences $\{C(\lambda_k)\}_{k=1}^{N'}$ and $\{\tau_{\lambda_k}\}_{k=1}^{N'}$ .
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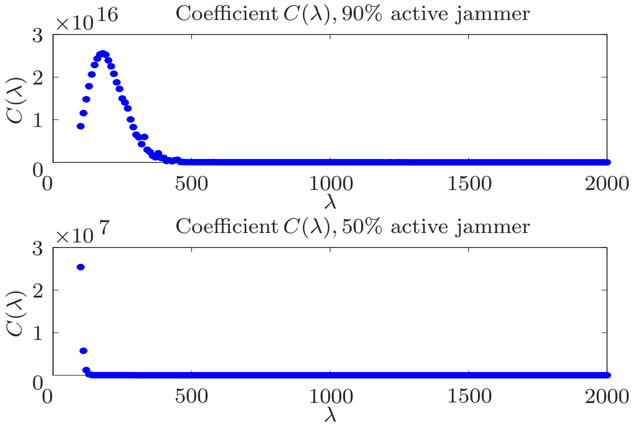

Fig. 6.1: *Coefficient* $C(\lambda)$ *under* 90% *and* 50% *active jammers*

6.1. Known jammer scenario. We consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \,,$$

$$u = \begin{bmatrix} -\lambda^3 + 3, -3\lambda^2 + 2, -3\lambda - 3 \end{bmatrix} x \,, \tag{6.1}$$

with its only eigenvalue at $-\lambda$, with algebraic and geometric multiplicity of 3, and 1, respectively; see Lemma 3.1. The only linearly independent eigenvector is given by solving the equation $(A+BK_\lambda+\lambda I)v_1 = 0$ for v_1. After some algebraic manipulations, we obtain, $v_1 = [1, -\lambda, \lambda^2]^\top$, while the other two generalized eigenvectors are $v_2 = [\frac{2}{\lambda}, -1, 0]^\top$, $v_3 = [\frac{3}{\lambda^2}, -\frac{1}{\lambda}, 0]^\top$. Hence, $T_\lambda = [v_1, v_2, v_3]$.

In order to perform a first set of simulations following Algorithm 1, we have chosen $\sigma = 0.1$, a more active and periodic jammer with $T = 1\,\text{sec}$, $T_{\text{on, }1}^{\text{cr}} = 0.9T$, $T_{\text{off, }1}^{\text{cr}} = 0.1T$, and a less active periodic jammer with $T_{\text{on, }2}^{\text{cr}} = 0.5T$, $T_{\text{off, }2}^{\text{cr}} = 0.5T$. In order to assess the analysis stated in Theorem 4.2, we have chosen the sequence $\{\lambda_k = 10k\}_{k=1}^{200}$. The corresponding $C(\lambda_k)$ sequences for each type of jammer and this example is shown in Figure 6.1, which confirms $\lim_{\lambda\to\infty} C(\lambda) = 0$. The asymptotic stability

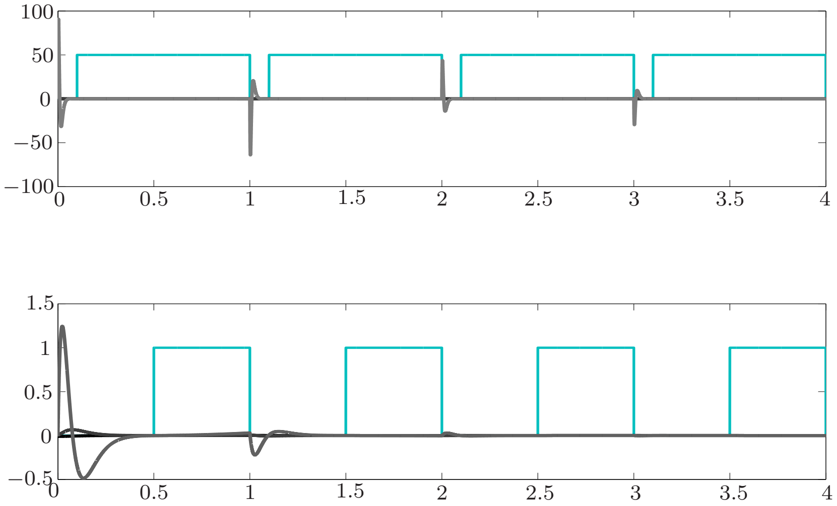

Fig. 6.2: *Temporal evolution of $x_i(t)$, $i = 1, 2, 3$, demonstrating the stability despite DoS signals. (Scaled) PWM jamming signals are superimposed in both plots.*

of System (6.1) under both types of jammers is guaranteed according to the values of $C(\lambda)$ corresponding to $\bar{\lambda}_{50\%} = 210$, and $\bar{\lambda}_{90\%} = 1360$, respectively, along with the resilient triggering strategy (3.7). The temporal evolution of the states showing stability is shown in Figure 6.2. From here, one can conclude that the stability of the system is guaranteed via Algorithm 1 despite the presence of very active jammers.

However, even though $C(\lambda)$ tends to zero, due to its increasing character, a large value of λ is required to guarantee a decrease in the state. Further simulations confirm that these are conservative values in general. Indeed, and for the sake of comparison with the algorithms in Section 6.2, let us consider a jammer with $T = 10\,\text{sec}$ and that is initially active from $t = 0$ until $t = 0.2\,\text{sec}$, and which after that behaves periodically with $T_{\text{on}}^{\text{cr}} = 0.4T$, and $T_{\text{off}}^{\text{cr}} = 0.6T$. One can observe that using $\lambda = 1.6 > \frac{1}{2} + \|N\| \approx 1.5$, with the corresponding $\tau_\lambda \approx 9.1 \times 10^{-3}$, under the strategy in (3.7), the system becomes unstable, see Figure 6.3 (left). When there is no jammer, this is a value of λ that is sufficient to stabilize the system following the approach in Remark 3.4, and which induces a τ_λ of similar magnitude to those reported in [28]. However, for this jammer and system, a $\tau_2 = 0.0058$ (with $\lambda = 2$) is enough to beat this jammer; and the performance increases significantly for $\tau_3 = 0.0023$ and $\lambda = 3$, see Figure 6.4 for the stability results.

Furthermore, following Proposition 3.5, we plot the evolution of τ_λ in Figure 6.3 (right). The figure shows that τ_λ indeed decreases to zero as function of λ in a rather fast manner and almost monotonically. This confirms the theoretical result, and can be indicative that large λ for this system may not be necessary.

6.2. Unknown jammer scenario: JAMCOID for Periodic Signals algorithm. For System (6.1), and under the JAMCOID FOR PERIODIC SIGNALS algo-

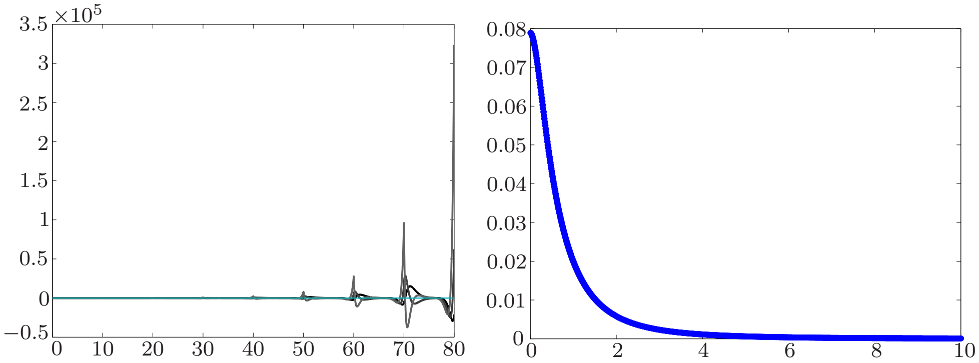

Fig. 6.3: *(Right) Temporal evolution of $x_i(t)$, $i = 1, 2, 3$, under Algorithm 1 for $\lambda = 1.6$. The (scaled) PWM jamming signal is also plotted. (Left) Evolution of τ_λ as a function of λ.*

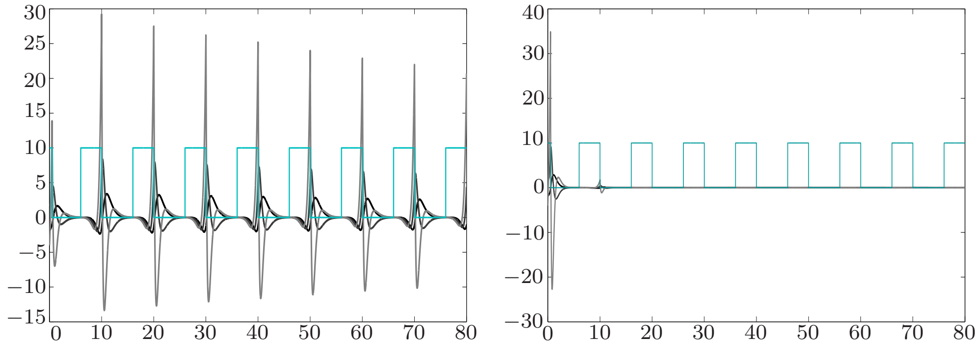

Fig. 6.4: *Temporal evolution of $x_i(t)$, $i = 1, 2, 3$, under Algorithm 1 for $\lambda = 2$ (left) and $\lambda = 3$ (right). The (scaled) PWM jamming signal is plotted in both figures.*

rithm, an active jammer after $t = 0.3\,\mathrm{sec}$ does not create a significant disturbance, and, similarly, a small jamming activity creates a large disturbance initially. Thus, we consider a jammer that is active from $t = 0$ until $t = 0.2\,\mathrm{sec}$, introducing a small clock asynchrony, and take $T = 10\,\mathrm{sec}$, $T_{\mathrm{on}}^{\mathrm{cr}} = 0.4T$, $T_{\mathrm{off}}^{\mathrm{cr}} = 0.6T$. To implement the algorithm, we take $\sigma = 0.1$, and the initial values $\hat{T}_{\mathrm{on}}^{\mathrm{cr}} = 7.8\,\mathrm{sec}$, $\hat{T}_{\mathrm{off}}^{\mathrm{cr}} = 4.2\,\mathrm{sec}$. Recall that the JAMCOID for Periodic Signals algorithm builds on Algorithm 1 to learn the jamming signal's parameters, adapt the choice of λ, and reduce communications to the off periods. A zoomed-in plot of an evolution of the state under the algorithm is shown in Figure 6.5. The adjustment of τ_λ is given in Figure 6.6 (left). Thus, starting with a smaller τ_λ, the JAMCOID for Periodic Signals algorithm changes it when obtaining better estimates of $\hat{T}_{\mathrm{on}}^{\mathrm{cr}}$, and $\hat{T}_{\mathrm{off}}^{\mathrm{cr}}$. For comparison on the size, note that, when there is no jammer, the system can be stabilized as in Remark 3.4 with a small $\lambda \approx 1.6 > \frac{1}{2} + \|N\| \approx 1.5$, and $\tau_\lambda \approx 9.1 \times 10^{-3}$ (similar to the scales seen in [28]). Thus, using the bound $C(\lambda)$ to deal with this jammer leads to a reduction of τ_λ by three orders of magnitude.

To show how the JAMCOID for Periodic Signals algorithm estimates the jam-

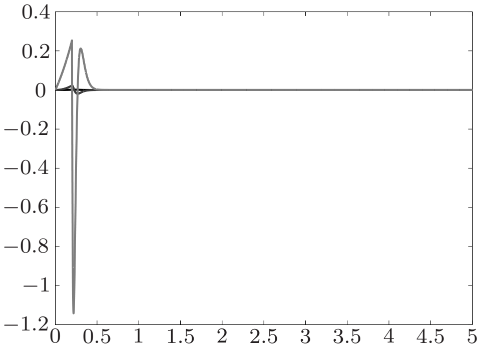

Fig. 6.5: *Temporal evolution of* $x_i(t)$*,* $i = 1, 2, 3$*, under the* JAMCOID FOR PERIODIC SIGNALS *algorithm.*

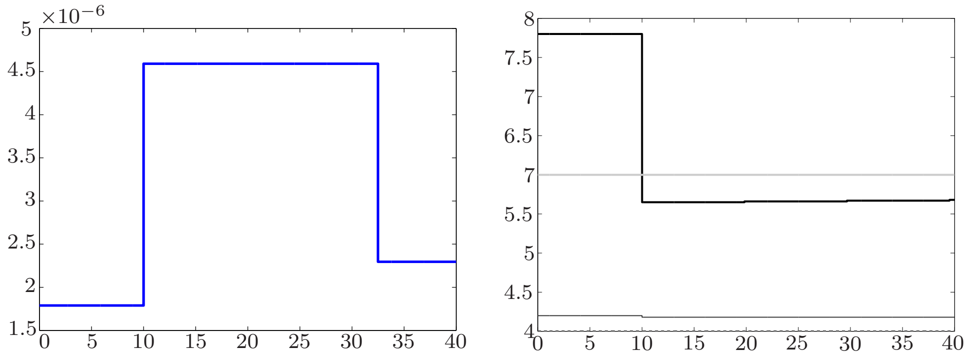

Fig. 6.6: *(Left) Temporal evolution of* τ_λ *under the* JAMCOID FOR PERIODIC SIGNALS *algorithm. (Right) Temporal evolution of the estimates of the jamming signal's parameters (t in seconds on the horizontal) under the* JAMCOID FOR PERIODIC SIGNALS *algorithm. The plot shows* $T^{\text{cr}}_{\text{on}}$ *(overlapping with the horizontal axis),* $T^{\text{cr}}_{\text{off}}$ *(light grey line at* $y = 7$*),* $\hat{T}^{\text{cr}}_{\text{on}}$ *(black line), and* $\hat{T}^{\text{cr}}_{\text{off}}$ *(grey line between* $y = 4.5$ *and* $y = 4$*).*

ming signal's parameters—a feature not present in Algorithm 1—we use a larger $\tau = 10^{-2}$ with a $\lambda = 1.6$. Recall that, in this algorithm, the stabilization strategy builds upon the estimation and synchronization strategy, and that the latter always works for any τ even though the stabilization may fail. Figure 6.6 (right) shows the results, starting from $\hat{T}^{\text{cr}}_{\text{on}} = 7.8\,\text{sec}$, $\hat{T}^{\text{cr}}_{\text{off}} = 4.2\,\text{sec}$. The plot illustrates how $\hat{T}^{\text{cr}}_{\text{on}}$ is a conservative upper bound of the true $T^{\text{cr}}_{\text{on}} = 4$ sec, and better adjusted at $t = 10\,\text{sec}$, and that $\hat{T}^{\text{cr}}_{\text{off}}$ remains a conservative lower bound of the true $T^{\text{cr}}_{\text{off}} = 6$ sec. As can be seen, the estimates are slightly adjusted after each new period to account for synchronization. The plot also shows that, starting from $\hat{T} = 12\,\text{sec}$, eventually $\hat{T} = \hat{T}^{\text{cr}}_{\text{off}} + \hat{T}^{\text{cr}}_{\text{on}} = 4.3$ sec $+5.8$ sec ≈ 10 sec $= T$.

6.3. Unknown jammer scenario: JAMCOID algorithm using an event-triggered implementation. Here, we implement the JAMCOID algorithm on System (6.1) but using an event-triggered strategy as described in Remark 5.3 (the re-

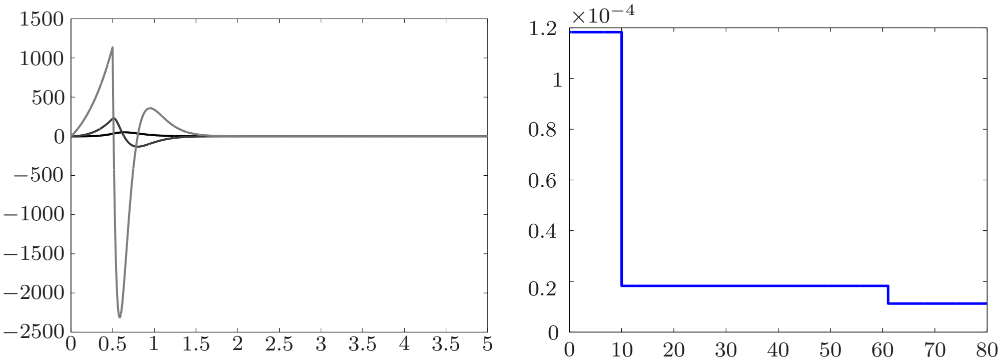

Fig. 6.7: *(Left) Temporal evolution of $x_i(t)$, $i = 1, 2, 3$, under the* JAMCOID *algorithm. (Right) Temporal evolution of τ_λ under the* JAMCOID *algorithm.*

sulting behavior using the time-triggered version is very similar.) We consider a jamming signal that first acts from $t = 0$ until 0.5 sec, and after that $T_{\text{off, }1} = 6$ sec, $T_{\text{on, }1} = 4$ sec, $T_{\text{off, }2} = 7.8$ sec, $T_{\text{on, }2} = 3.2$ sec, $T_{\text{off, }3} = 10.4$ sec, $T_{\text{on, }3} = 5.12$ sec, $T_{\text{off, }4} = 0.1$, $T_{\text{on, }4} = 0.12$, and $T_{\text{off, }5} = 0.12$. For the algorithm implementation, we choose $\sigma = 0.1$ and the initial estimates $T_{\text{on}}^{\text{cr}} = 2.2$ sec, $T_{\text{off}}^{\text{cr}} = 8.8$ sec. Recall that the JAMCOID algorithm builds on Algorithm 1 to learn the jamming signal's parameters, but unlike the JAMCOID for Periodic Signals algorithm, the adaptation of λ is always reduced to cope with the worst-case jamming scenario, and communications are always active. Figure 6.7 (left) shows the stabilization of the state under the JAMCOID algorithm. Even though the initial condition is taken to be the same as that of the simulation in Section 6.2, the system is subject to an initial jamming signal that acts for more than double the time, resulting in a much larger disturbance. For implementation, we used the λ prescribed by the bound $C(\lambda)$ and based on the estimates of the worst-case $\hat{T}_{\text{on}}^{\text{cr}}$ and $\hat{T}_{\text{off}}^{\text{cr}}$ as in Remark 5.3.

In Figure 6.7 (right), we plot the evolution of the lower bound τ_λ for the event triggers, while Figure 6.8 (left) plots the differences between two consecutive events and the occurrences of such events. The latter shows the irregularity of the control update times as compared with the periodic case. Similarly to the simulation in the previous section, these times are of order 10^{-6}, lower by two orders of magnitude than the τ required in the jammer-free case. Unlike what happens for the JAMCOID for Periodic Signals algorithm, the τ_λ and the inter-event times in JAMCOID never grow back to larger values, because the operator is reducing its estimate of $\hat{T}_{\text{off}}^{\text{cr}}$ to the lowest and worst-case value found. Finally, and similarly to Section 6.2, Figure 6.8 (right) shows the estimates of the jammer signal's parameters using larger inter-sampling times of order 10^{-2}. Around $t = 10\,\text{sec}$, $\hat{T}_{\text{off}}^{\text{cr}}$ is adjusted to be a bit smaller than the $T_{\text{off},1}^{\text{cr}} = 4\,\text{sec}$, while $\hat{T}_{\text{on}}^{\text{cr}}$ is increased a bit over $T_{\text{on},1}^{\text{cr}} = 6\,\text{sec}$. Even though after this time, there is no jammer over a longer period, JAMCOID maintains $\hat{T}_{\text{off}}^{\text{cr}}$ to the worst-case value. The value of $\hat{T}_{\text{on}}^{\text{cr}}$ is then adjusted after the operator finds a longer on period between $t = 36\,\text{sec}$ and $t = 60\,\text{sec}$, to $\hat{T}_{\text{on}}^{\text{cr}} = 5.12\,\text{sec}$.

7. Conclusions and Future Work.

In this paper, we have considered controllable single-input continuous linear systems subject to power-constrained PWM DoS

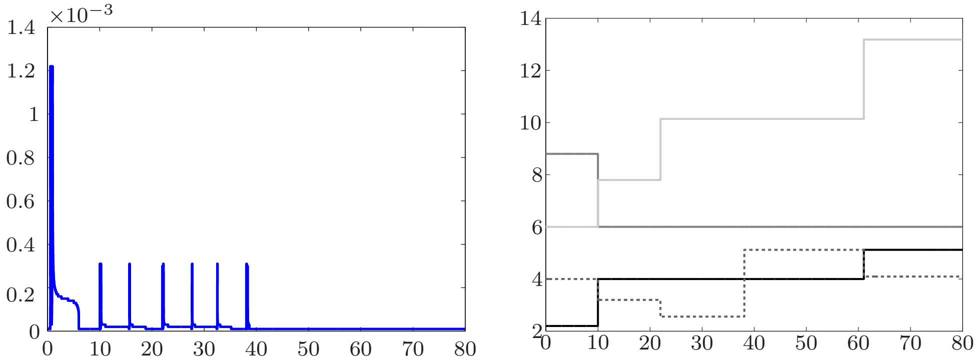

Fig. 6.8: *(Left) Plot of the event-triggering time differences for the simulation run of the* JAMCOID *algorithm. (Right) Temporal evolution of the estimates of the jamming signal's parameters under the* JAMCOID *algorithm.The plot shows* $T_{\rm on}^{\rm cr}$ *(dotted grey line below* $y = 6$*),* $T_{\rm off}^{\rm cr}$ *(increasing light grey line from* $y = 6$ *to below* $y = 14$*),* $\hat{T}_{\rm on}^{\rm cr}$ *(black line between* $y = 2$ *and* $y = 6$*), and* $\hat{T}_{\rm off}^{\rm cr}$ *(decreasing grey line between* $y = 10$ *and* $y = 6$*).*

jamming signals. We have proposed a resilient parameter-dependent control and triggering strategies in three different problem scenarios which guarantee system stability under different assumptions on the knowledge of the jamming signal. The functionality of the theoretical results entailing both partially known and unknown DoS signals has been demonstrated in a simulation environment.

There are several questions that we would like to address in future work. First, there is the question of how to obtain less conservative bounds for λ that can guarantee system stability. Second, how to extend the results to nonlinear controllable systems. Nonlinearities and the initial system condition will play a role in the definition of the appropriate control laws. In addition, one would have to devise appropriate offline motion planning algorithms for under-actuated systems in order to maintain the system under control during the on periods. Second, although we have also studied a PWM DoS signals characterized by a deterministic sequence $\{T^n\}$ with variable time-intervals, an intriguing question would be to devise stochastic control strategies to deal with a possible stochastic behavior of the jammer.

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