289 B: Mathematical Analysis for Applications (Winter 2025)

Instructor: Sonia Martínez, FAH 3302, 858-822-4243, soniamd@ucsd.edu

Lecture Time and Place: Tu - Th 9:30am - 11:00am,

Office Hours: Tues 3:15pm to 4:15pm, location TBA

Texts: The following are some references for the course, although any book in mathematical analysis should be useful. Notes taken in class will provide a concise material for the course (supplemented with notes): *Measure and Integral.* M. Brokate and G. Kerstin. Birkhauser, 2015.

Prerequisites: 289A or equivalent knowledge about real and complex number systems, basic topology (open, closed, neighborhood, compact, metric spaces, connected sets) and continuity.

Course Objectives: This course is the second one of a series of three, covering miscellaneous topics of mathematical reasoning, mathematical analysis for real/vector-valued functions of one and/or several variables. Currently, the level of mathematics necessary for a successful path through much of the MAE PhD graduate curriculum is above that with which students typically arrive. The goal of this sequence is to bring you to the point where you can more easily handle the material in more advanced courses of the program. While doing so, we will aim to present applications of the theory to dynamics systems, optimization, and control. A special emphasis will be placed on proof techniques. The course is math intensive does not cover linear algebra in detail, this is done through courses such as 280a and 280b. Finally, note that if you are enrolled in the new MS program, you may not need to take this course. (Of course, you still may take it if you like.)

The topics to be covered in this particular class refer to measure, integration with main applications to probability. The tentative outline of the class is the following (time permitting):

- Measurable sets and measurable functions. Random variables.
- Measures. Probability distributions, independent random variables.
- Integral of nonnegative functions. Monotone convergence theorems. The Markov inequality.
- Integrable functions. The dominated convergence theorem. Holder's and Jensen's inequalities. Relationship to Riemann's integral. Integration wrt probability distributions. Expectation of a random variable. The characteristic function.
- Uniqueness and regularity of measures.
- Multiple integrals and product measures. Fubini's theorem. Joint distributions. Conditional probability. Bayes' theorem.
- Absolute continuity. Functions of bounded variation. Signed measures.
- Spaces of integrable functions (L^p spaces). Moments, independence, conditional expectation.
- Densities and conditioning. Radon-Nikodym theorem. Riesz Representation theorem. Conditional expectation. Martingales.
- (Partially, time permitting). Convergence in the mean and convergence in the measure (or in probabilility). Relationship between the two. The Chebyshev's inequality. Weak law of large numbers. The Borel Cantelli's lemmas. The strong law of large numbers. Weak convergence and Prokhorov's theorem. The central limit theorem.

Course website:

http://muro.ucsd.edu/sonia/teaching/mae289b-W2025/

The canvas website is https://canvas.ucsd.edu/courses/61898

Class participation: Attendance is required and class participation very much encouraged.

Grading: The final course grade will be calculated as follows:

- Homework: 100%

Homework will be assigned almost every week (maximum of 7 to 8 assignments this quarter) that you will have to turn in. However, not all homework assignments and/or problems will be corrected for the final grade. The main goal of the homework is that you practice what is covered in class.

The homework problems can be solved with others (maximum of 3 people), but an independent solution must be turned in by each student. Please include in your assignment the names of students you did your homework with. It is essential that all assignments for this course be completed in accordance with the precepts of the Code of Academic Integrity. Failure to comply with the Code of Academic Integrity will not be tolerated.