MAE40 Linear Circuits (for non-electrical engs)

Topics covered

Circuit analysis techniques

Kirchoff's Laws – KVL, KCL

Nodal and Mesh Analysis

Thévenin and Norton Equivalent Circuits

Resistive circuits, RLC circuits

Steady-state and dynamic responses

Impulse and step responses   Laplace transforms

Sinusoidal steady-state response

Circuit design techniques

Active circuit elements – dependent sources and operational amplifiers

Feedback basics

Signal filtering

What do I expect you to know?

Prerequisites

Mathematics 21D or 20D (some are doing this concurrently)

Solution of sets of linear equations

$Ax=b$ equations for vector x given matrix A and vector b

Solution of constant coefficient linear ordinary differential equations

Laplace transform

Initial conditions and forced response

Complex analysis

Numbers, arithmetic, magnitudes and phases, poles, zeros

Physics 2B

Behavior of circuit elements: R, L and C

Underlying physical principles

Why should you be excited about MAE40?

Linear circuits are exceptionally well described by simple and powerful mathematics

The quality of concordance between measured physical behavior and mathematical description is amazing
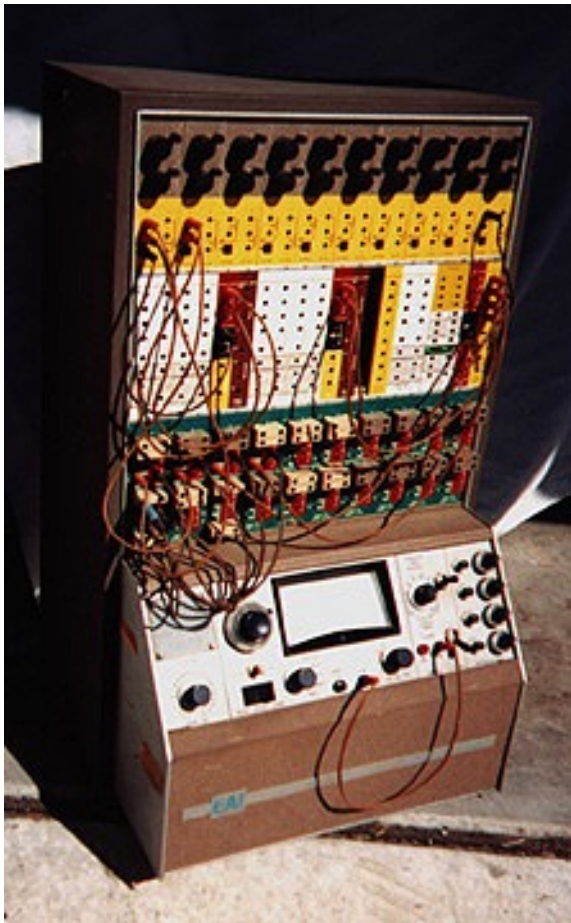
This is an analog computer from around 1963
It was used to solve nonlinear ODEs via analog circuitry
It contained: about 20 integrators based on valves or transistors, some nonlinear function blocks using diodes, comparators
This was the state of the art for much physical system simulation
It relies on ideas underpinning MAE40
The tools of MAE40 are immediately useful in design

Why should you still be excited about MAE40?

The *front and back ends* of your digital cell phone are comprised of analog circuits

    This is pretty much true of all *digital* technology

Why does the stagecoach wheel appear to rotate backwards?

    Aliasing

        A high frequency masquerading as a low frequency

    Anti-aliasing filters MUST be used in all

    sampled data systems

After MAE40 you will be able to start designing

    such anti-aliasing filters

    The real thing!

Why not leave all circuit stuff to electrical engineers?
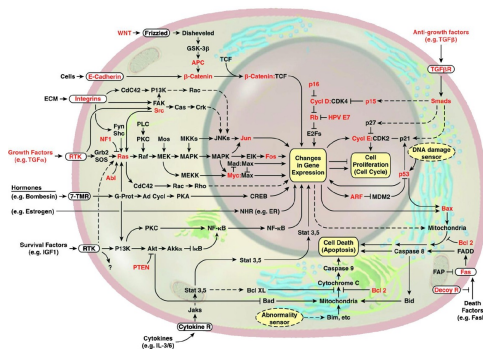
- Sensors are essential components of many (MAE) devices
- Sensors are based on physics
- Therefore, MAEs often responsible for:
  - Sensor selection
  - Sensor noise and sampling effects
  - Power needs for driving motors
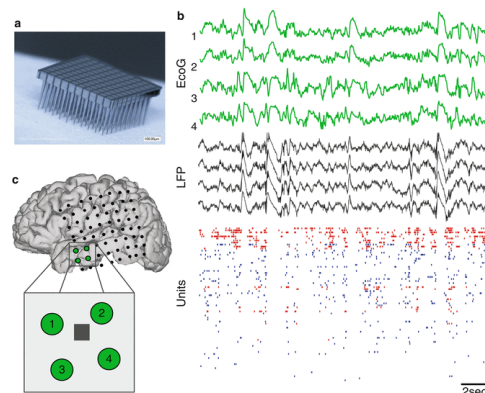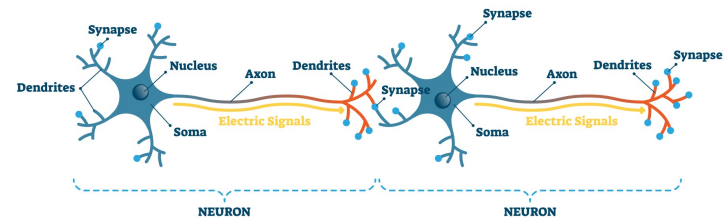- Circuits are all around us… and inside!

Onewheel

Pitot Tube on Aircraft

MAE40 Linear Circuits
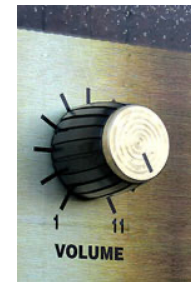Biological circuits

Local Field Potentials

Synaptic current

Why should you still be excited about MAE40?

You will learn how these knobs and plugs work:

We will not have time to cover this one:

Also, check out cool video of SensorLab created by Porf. Delson at https://www.youtube.com/watch?v=KxgkVhqwgFo

Why should you still be excited about MAE40?

The Guardian news article screenshot.

US nearly detonated atomic bomb over North Carolina – secret document

Exclusive: Journalist uses Freedom of Information Act to disclose 1961 accident in which one switch averted catastrophe

Ed Pilkington in New York
The Guardian, Friday 20 September 2013 12.03 EDT

The bomb that nearly exploded over North Carolina was 260 times more powerful than the device which devasted Hiroshima in 1945. Photo: Three Lions/Getty Images

A secret document, published in declassified form for the first time by the Guardian today, reveals that the US Air Force came dramatically close to detonating an atom bomb over North Carolina that would have been 260 times more powerful than the device that devastated Hiroshima.

The document, obtained by the investigative journalist Eric Schlosser under the Freedom of Information Act, gives the first conclusive evidence that the US was narrowly spared a disaster of monumental proportions when two Mark 39 hydrogen bombs were accidentally dropped over Goldsboro, North Carolina on 23 January 1961. The bombs fell to earth

"The bombs fell to earth after a B-52 bomber broke up in mid-air, and one of the devices behaved precisely as a nuclear weapon was designed to behave in warfare: its parachute opened, its trigger mechanisms engaged, and only **one low-voltage switch** prevented untold carnage."

"… a senior engineer in the Sandia national laboratories responsible for the mechanical safety of nuclear weapons concludes that 'one simple, dynamo-technology, **low voltage switch** stood between the United States and a major catastrophe.'"

MAE40 Linear Circuits

Low Voltage Switches

GE RFS-6 Discontinued   GE RS2-32 Current Model
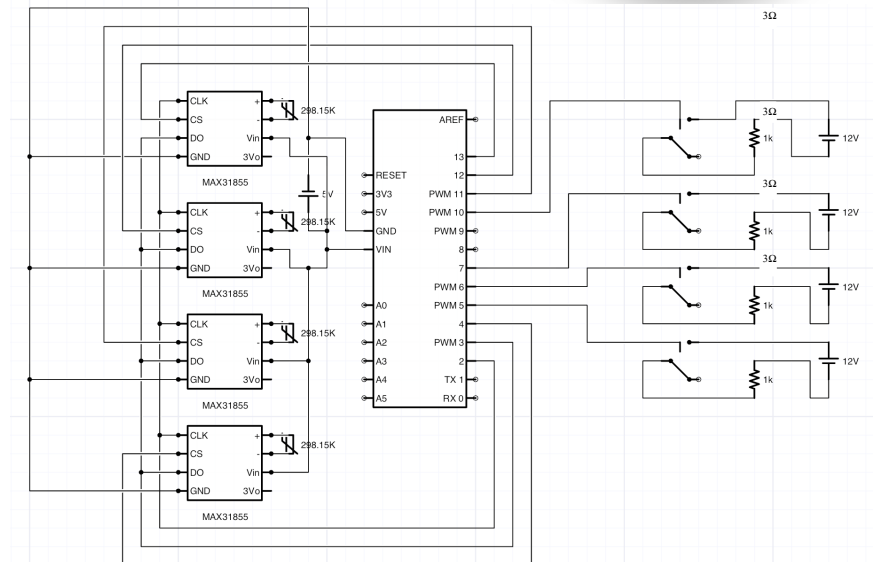
Our TA: Brooklyn Asai

- **Electronics useful in research**
  - Study of heated liquid films
  - Arduino, power supplies, relays, thermocouples, amplifiers, etc.
- Spec sheets are important!
- Many resources for circuit design (iCircuit, Arduino, Seeed, Fritzing) before you build

Real Engineers Use Spec. Sheets

A Specification (Spec.) sheet defines what the manufacturer guarantees

- Electrical input
- Electrical output
- Mechanical properties
- Life and durability

A hobbyist may get a circuit to work through trial and error, but it may not work consistently.

We will use spec. sheets to ensure that our circuits are reliable.

MAE40 Linear Circuits

finder	65 Series - Power relays

Features	65.31	65.61

20 A Power relays
1 NO + 1 NC (SPST-NO + SPST-NC)

65.31 Flange mount
(Faston 250 connections)
65.61 PCB mount
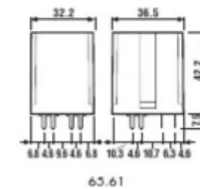
- AC coils & DC coils
- Cadmium Free option available

- 20 A rated contacts
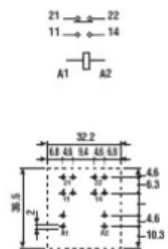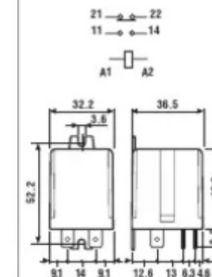- Flange mount/Faston 250 (6.3x0.8 mm) connection

- 20 A rated contacts
- PCB mount - bifurcated terminals

65.61

* With the AgSnO₂ material the maximum peak current is 120 A - 5 ms on NO contact.

Contact specification			
Contact configuration		1NO+1NC (SPST-NO+SPST-NC)	1NO+1NC (SPST-NO+SPST-NC)
Rated current/Maximum peak current	A	20/40*	20/40*
Rated voltage/Maximum switching voltage	V AC	250/400	250/400
Rated load AC1	VA	5,000	5,000
Rated load AC15 (230 V AC)	VA	1,000	1,000
Single phase motor rating (230 V AC)	kW	1.1	1.1
Breaking capacity DC1: 30/110/220 V	A	20/0.8/0.5	20/0.8/0.5
Minimum switching load	mW (V/mA)	1,000 (10/10)	1,000 (10/10)
Standard contact material		AgCdO	AgCdO
Coil specification			
Nominal voltage (U_N)	V AC (50/60 Hz)	6 - 12 - 24 - 48 - 60 - 110 - 120 - 230 - 240 - 400	
	V DC	6 - 12 - 24 - 48 - 60 - 110 - 125 - 220	
Rated power AC/DC	VA (50 Hz)/W	2.2/1.3	2.2/1.3
Operating range	AC	(0.8...1.1)U_N	(0.8...1.1)U_N
	DC	(0.85...1.1)U_N	(0.85...1.1)U_N
Holding voltage	AC/DC	0.8 U_N/0.6 U_N	0.8 U_N/0.6 U_N
Must drop-out voltage	AC/DC	0.2 U_N/0.1 U_N	0.2 U_N/0.1 U_N
Technical data			
Mechanical life AC/DC	cycles	10 · 10⁶/30 · 10⁶	10 · 10⁶/30 · 10⁶
Electrical life at rated load AC1	cycles	80 · 10³	80 · 10³
Operate/release time	ms	10/12	10/12
Insulation between coil and contacts (1.2/50 µs)	kV	4	4
Dielectric strength between open contacts	V AC	1,500	1,500
Ambient temperature range	°C	–40...+75	–40...+75
Environmental protection		RT I	RT I

Circuit variables (T&R Chap 1)

Charge and Energy

-1 coulomb (C) = 6.25×10^{18} electrons' charge

1 ampere (A) = 1 coulomb/second

$$i = \frac{dq}{dt}$$

Current is a measure of charge passing

Direction of flow is of positive charge

This is important for understanding some devices

Voltage/potential difference/electromotive force/tension

Voltage measures the energy gained by a charge

1 joule(J)/coulomb = 1 volt (V)

$$v = \frac{dw}{dq}$$

Voltage is measured between two points (potential difference)

This is akin to gravitational potential and fluid flow or temperature and heat flow

Circuit variables (contd)

Power is the rate of energy change per unit time $\quad p = \dfrac{dw}{dt}$ watts (W)

Note the chain rule $\qquad p = \left(\dfrac{dw}{dq}\right)\left(\dfrac{dq}{dt}\right) = vi$

The instantaneous electrical power associated with a voltage and a current flow is given by the product $v \times i$
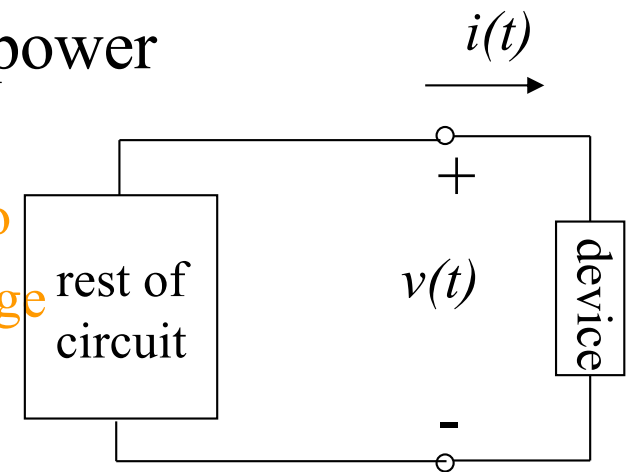
Passive sign convention

Power is positive when a device absorbs power

Signs are measured as illustrated

"The current variable must be defined so positive current enters the positive voltage terminal of the device"

Note that $v(t)$ or $i(t)$ can be negative

In the illustration the current into the device is the negative of that into the rest of the circuit

An example

The figure at right depicts the circuit inside my flashlight

The devices are the battery and the lamp
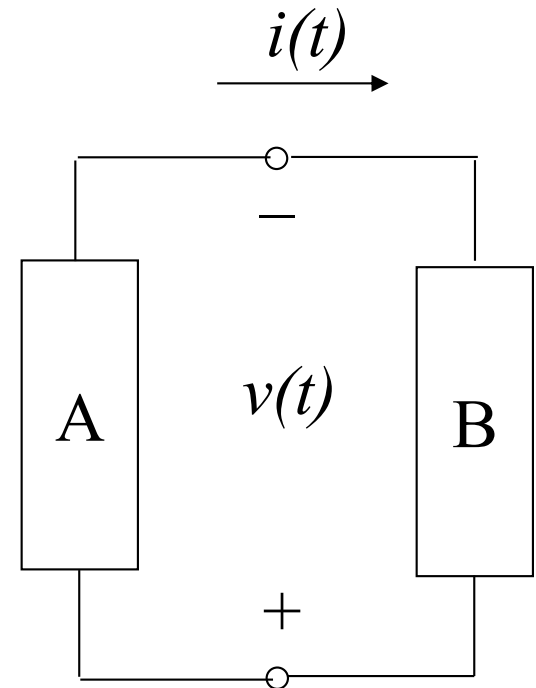
A voltmeter reads *v(t)* is 12V

An ammeter reads *i(t)* is −1.5A

Which device is the battery and which is the lamp? How much power is being consumed by the lamp?

Using the passive sign convention

The power absorbed by device A is 12×-1.5 W = -18 W

A is the battery and B is the lamp, which consumes 18W

$i(t)$

A $v(t)$ B

−

+

Importance of Power in ME Systems
Power = torque x rpm

Car Transmission

Large metal gears

Typical gear ratio: 1:5

Power: Over 200 horsepower

Toy Transmission

Small plastic gears

Typical gear ratio: 1:5-100

Power: tiny fraction of a horsepower

Clearly gear ratio is not sufficient to describe a transmission's performance. Power is key!

Importance of Power in EE Systems
Power = voltage x current

MAE 3 Power Supply

Output: 5VDC, 3A

P = 3 amp x 5 V =15 Watts

Note large heat sinks

MAE 170 Arduino Digital Output

Output: 5VDC, 20mA

P = 20 **milli**amp x 5 V

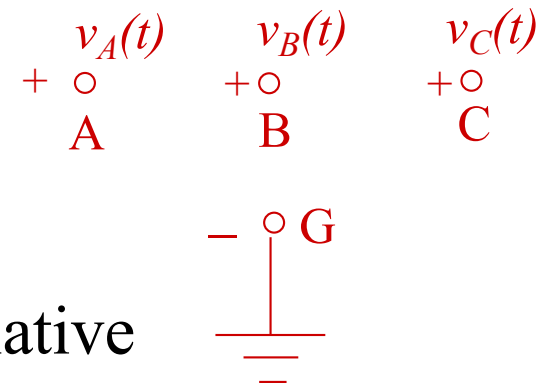   = 0.1Watts

Orders of magnitude less!

Circuit variables (contd)

Ground

Because all voltages are measured between two points, there is no absolute voltage - it is a potential

In circuits we often refer all voltages of points as being relative to a fixed point or *ground* voltage

$v_A(t)$ $v_B(t)$ $v_C(t)$

$+$ ○ $+$○ $+$○

A   B   C

This is depicted via the ___ symbol

$-$ ○ G

The terms v_A etc mean the voltage at *A* relative to *G*

This is like referring heights relative to sea level

Gravity is also a potential

Check out the elevator buttons in the Math & App Phys Building!

Circuit variables (contd)

A *circuit* is a collection of interconnected electrical devices

For us:

All electromagnetic interactions in the circuit take place within the devices (This is a *lumped-parameter* circuit)

The circuit devices are connected together by wires which are ideal

They have the same voltage at both ends instantaneously

They propagate current without loss instantaneously

They may be stretched arbitrarily without changing properties

All circuit devices have at least two terminals and are assumed not to accumulate charge – current in equals current out

A *node* is the junction of terminals of two or more devices

A *loop* is a closed path formed by tracing through an ordered sequence of nodes without passing through any node more than once

Circuit variables

The important physical circuit variables are currents into and voltages across circuit elements and their associated power consumption or production.

Circuit analysis consists of solving for the circuit variables in a given circuit

Circuit design or *synthesis* consists of constructing a circuit whose circuit variables exhibit a specified behavior

The *topology* of a circuit is the key structure

It can be stretched and wrapped around without changing circuit variables

Nodes can be stretched and shrunk to cover many terminals

Circuit Topology

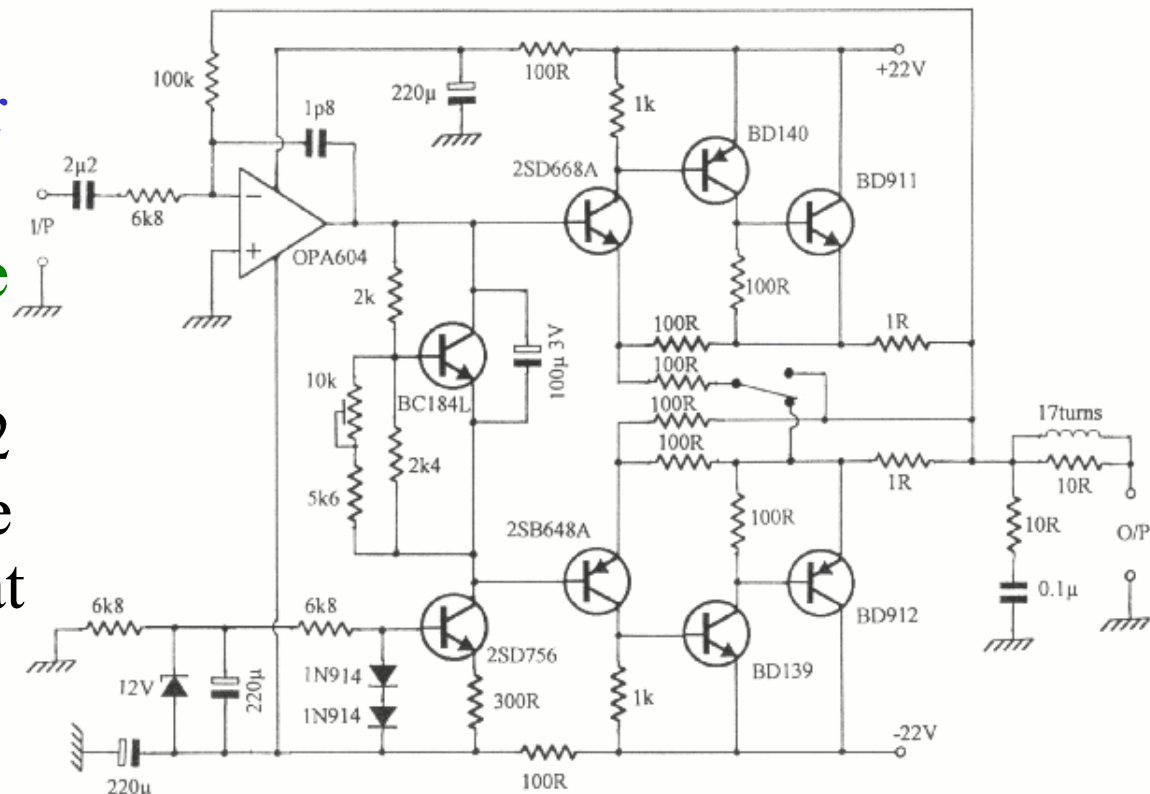This is the circuit diagram of an audio amplifier

It shows the relationship between parts but not their actual physical location

The ground/earth is likely the chassis or case

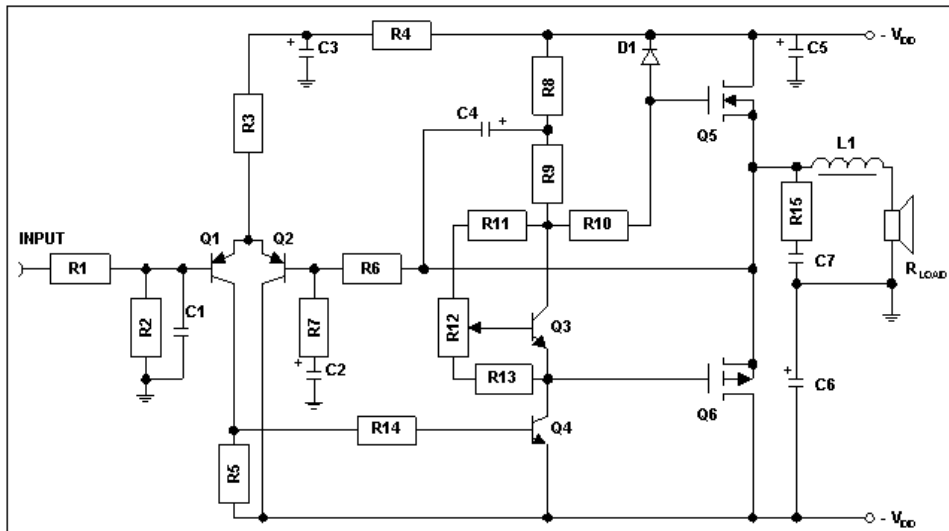The output transistors BD912 and BD911 will need to be attached to an external heat sink

The op-amp OPA604 will be board mounted

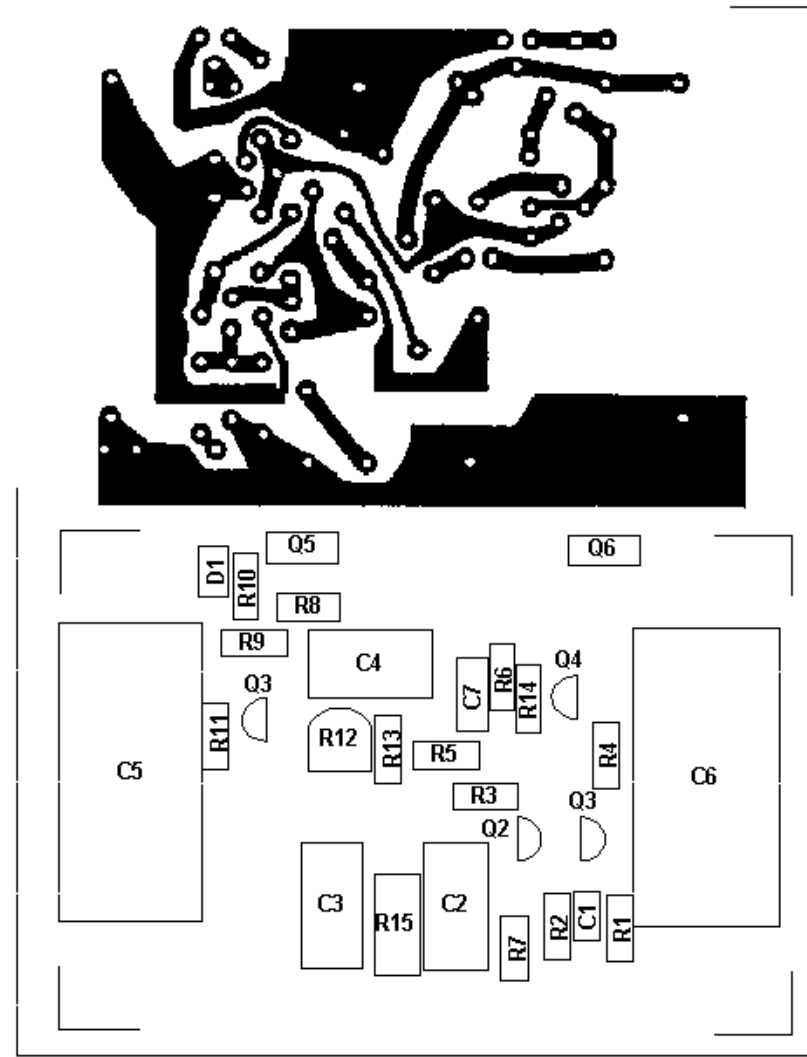The volume control needs to be accessible

Circuit Topology – high power amplifier with complementary HEXFETS

Circuit Diagram

Circuit Board Layout

Figure 2. Amplifier Printed Circuit Board Layout

HEXFET – power MOSFET
from International Rectifier

Basic Circuit Analysis (T&R Chap 2)

Kirchoff's Current Law (KCL)

The algebraic sum of currents entering a node is zero at every instant

> This is a restatement of the principle of conservation of charge

> Alternatively, but not so nice,

The sum of currents entering a node is equal to the sum of currents leaving a node at every instant

> KCL provides linear constraints between the currents in a circuit

In a circuit containing a total of N nodes there are only N-1 independent KCL connection equations
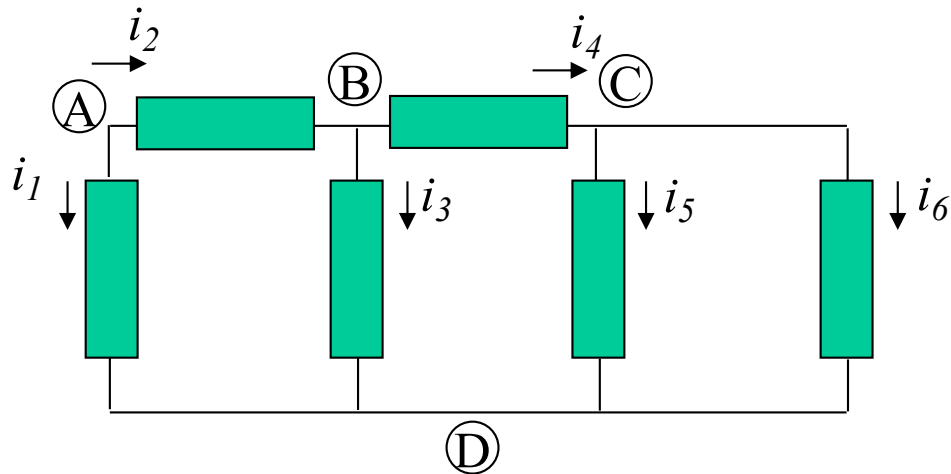
T&R, 5th ed, Exercise 2-1 p 22

Write KCL at nodes *A, B, C, D*

Node A: $\quad -i_1 - i_2 = 0$

Node B: $\quad i_2 - i_3 - i_4 = 0$

Node C: $\quad i_4 - i_5 - i_6 = 0$

Node D: $\quad i_1 + i_3 + i_5 + i_6 = 0$

If $i_1 = -1$mA, $i_3 = 0.5$mA, $i_6 = 0.2$mA, find the rest

$\quad i_2 = 1$mA

$\quad i_4 = 0.5$mA

$\quad i_5 = 0.3$mA

$$\begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
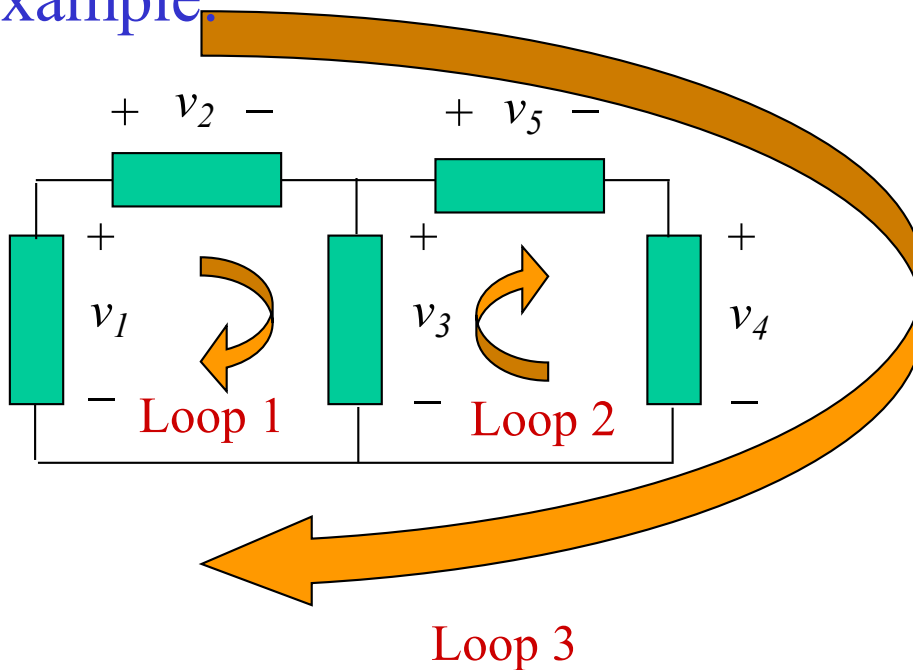
Note that we have 4 nodes

But only 3 independent equations

Kirchoff's Voltage Law (KVL)

The algebraic sum of voltages around a loop is zero at every instant

This is really a restatement of voltage as a potential, effectively a statement of the principle of conservation of energy

Example:

$$\text{Loop1}: -v_1 + v_2 + v_3 = 0$$

$$\text{Loop2}: -v_3 + v_4 + v_5 = 0$$

$$\text{Loop3}: -v_1 + v_2 + v_4 + v_5 = 0$$

$$\begin{pmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Circuit Analysis

One way to consider the analysis of a circuit (i.e. the computation of the currents in and voltage across the circuit elements) is through the specification of constraints on these variables

KCL and KVL provide one set of constraints on circuit variables reflecting connection topology

The i-v properties of the devices themselves provide further constraints

N nodes and E circuit elements

N-1 independent KCL relations

E-N+1 independent KVL relations (number of independent loops)

E independent i-v characteristics

$2E$ circuit variables

If we keep to linear devices then we have linear circuit analysis

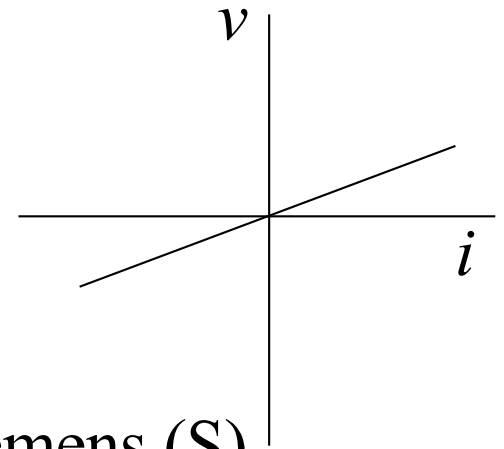$2E$ Linear algebraic equations for $2E$ variables

Linear Circuit Elements

Linearity in *v-i* relations

    Doubling *v* doubles *i* and vice versa $i=\alpha v$

        Straight lines in *i-v* plane

    Note that $i=\alpha v+\beta$ describes an *affine* relation

Resistor

    $v=Ri$ or $i=Gv$ Ohm's Law

    R resistance in ohms (Ω), *G* conductance in siemens (S)

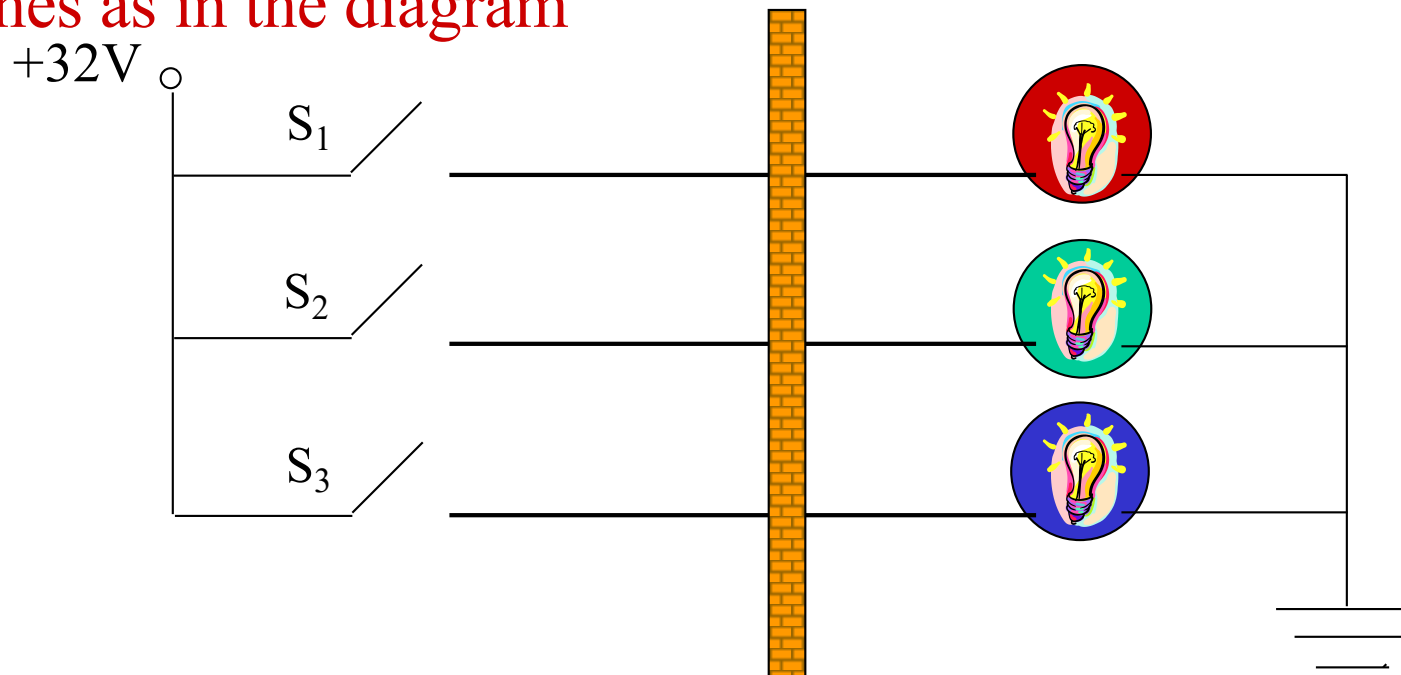   Power $P=i^2R= v^2/R=v^2G$

    A resistor always absorbs power – *i-v* line has positive slope

This is an instantaneous linear element

    We will look at circuit elements with memory like *C* and *L* later in the course

Time out puzzle

We have three lamps – red, green and blue – connected to three switches as in the diagram

+32V

S_1

S_2

S_3

The lamps are outside the switch room and we are alone

What single test can we perform by playing with the switches to determine which switch is connected to which light?
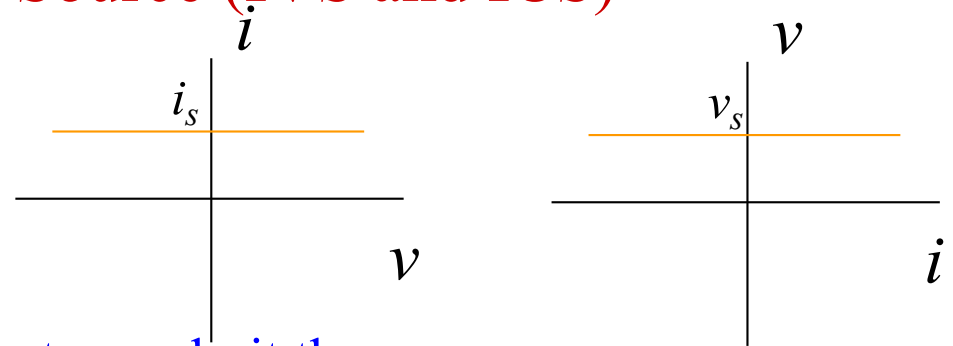
I know of two solutions, one of physicists, one of engineers

Linear Circuit Elements (contd)

Independent Voltage and Current Source (IVS and ICS)

IVS $v=v_s$ for any i

ICS $i=i_s$ for any v

Affine elements rather than linear, but we admit them

Most sources of electrical energy (batteries, generators, power systems) are modeled as voltage sources. Examples of current sources include diode and transistor current sources

Open circuit and short circuit – variants of ICS and IVS resp.

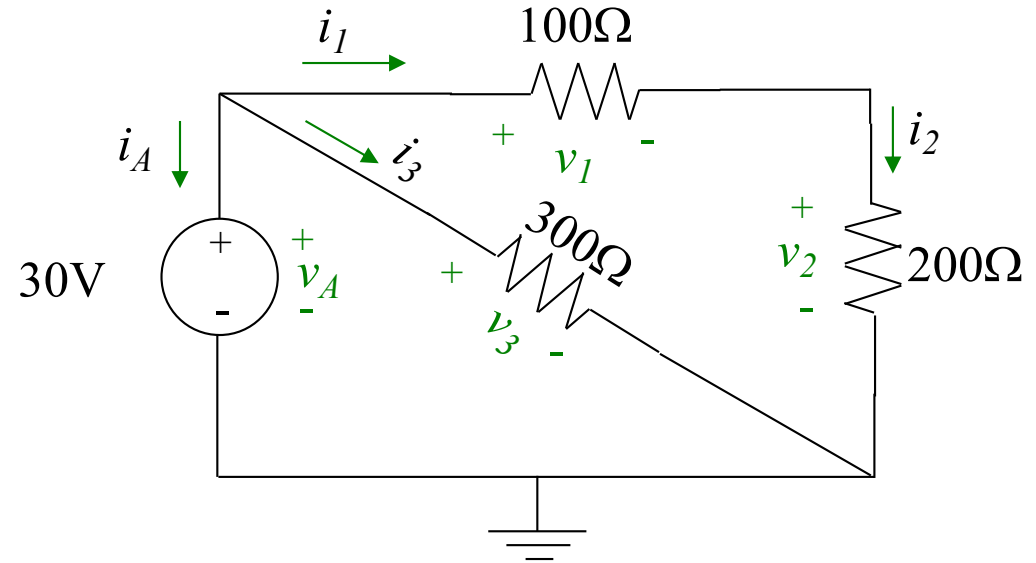Open circuit $i=0$ for any v Same as $R=\infty$

Short circuit $v=0$ for any i Same as $R=0$

These are truly linear since the curves pass through (0,0)

Switches – either o.c. or s.c.

Example 2-10 (T&R, 5th ed, p. 31)

Write a complete set of equations
  for each element

Write a complete set of connection equations

Solve these equations for all currents and voltages
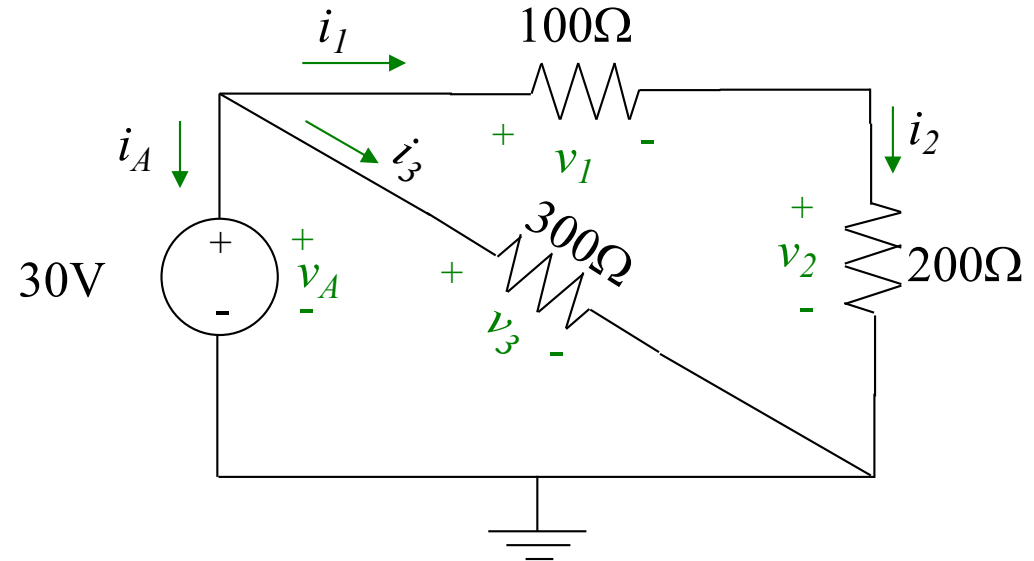
Example 2-10 (T&R, 5th ed, p. 31)

Write a complete set of equations
for each element

$v_A=30V;$

$v_1=100i_1;$

$v_2=200i_2;$

$v_3=300i_3;$

Write a complete set of connection equations

$-i_A-i_1-i_3=0; \quad i_1-i_2=0; \quad -30+v_3=0; \quad v_1+v_2-v_3=0;$

Solve these equations for all currents and voltages

$v_A=30V; \quad v_1=10V; \quad v_2=20V; \quad v_3=30V;$

$i_A=-200mA; \quad i_1=i_2=i_3=100mA;$
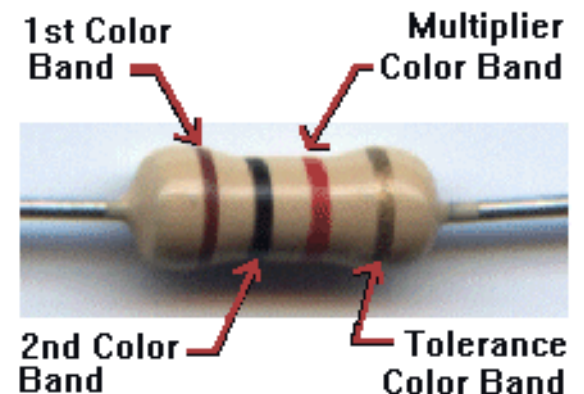
All you ever wanted to know about resistors…

Resistors are color coded to indicate their value

Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Grey	White
0	1	2	3	4	5	6	7	8	9

and tolerance

5%	10%	20%
Gold	Silver	Empty

This is a 1KΩ resistor with 5% tolerance

1st Color Band
2nd Color Band
Multiplier Color Band
Tolerance Color Band

Resistors come at standard values (e.g., at 10% tolerance)

1	1.2	1.5	1.8	2.2	2.7	3.3	3.9	4.7	5.6	6.8	8.2	Nominal
.9	1.08	1.35	1.62	1.98	2.43	2.97	3.51	4.23	5.04	6.12	7.38	Lower
1.1	1.32	1.65	1.98	2.42	2.97	3.63	4.29	5.17	6.16	7.48	9.02	Upper

Even more than you ever wanted to know about resistors …

The distribution of resistances

    Why should this be so?

How can we get a 258.3Ω resistor?
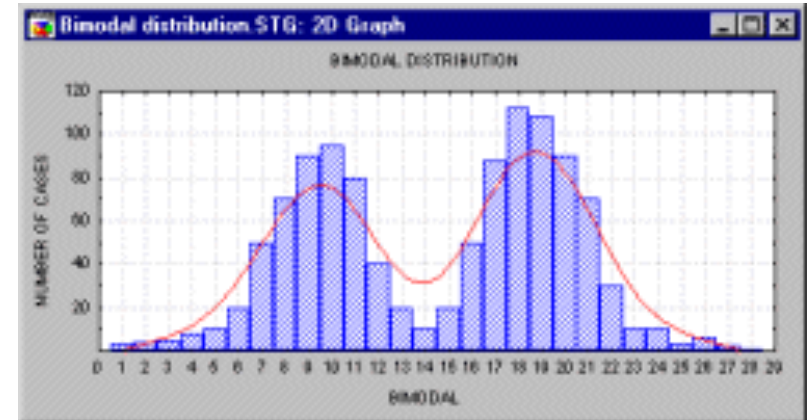
    Keep testing the ones in the box

    A really finely tunable potentiometer

       (variable resistor) – a *trimpot*

    A coarsely tunable potentiometer in parallel with a lower R

    Make one from high precision components

-10%   +10%

Nominal value

Another important characteristic of R is the power rating

    Why?

Example Problem:
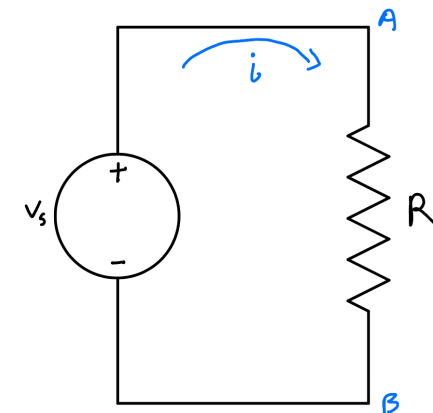# Voltage Output of Real World Power Supply

What will be the voltage output of the real world power supply for the following values of R

Real world Voltage Source

Will output 5 Volts of Direct Current **up to** 3 Am

R	Voltage Output
100 Ω	
10 Ω	
1 Ω	

Solution to Voltage Output

The voltage source will output the specified voltage as long as the current rating is not exceeded.

Current $i = V/R$

For $R = 100\,\Omega$ $\quad i = \frac{5}{100} = 0.05\,A \quad$ within spec ✓

For $R = 10\,\Omega$ $\quad i = \frac{5}{10} = 0.5\,A \quad$ within spec ✓

For $R = 1\,\Omega$ $\quad i = \frac{5}{1} = 5\,A \quad$ OUT OF SPEC!!!

Real world Voltage Source

Output: 5VDC, 3A

Voltage output of the real world power supply

R	Voltage Output
100 Ω	5V
10 Ω	5V
1 Ω	less than 5V, 0V, or fire!